Week 1 - Operations on Functions

We can combine functions to get new ones:

$(f+g)(x) = f(x) + g(x)$ sum

$(f-g)(x) = f(x) - g(x)$ difference

$(f \cdot g)(x) = f(x) \cdot g(x)$ product

$(f/g)(x) = f(x)/g(x)$ quotient

Practice

Find the sum, difference, product, and quotient of

$f(x) = x^2+1$ and $g(x) = x-2$

$(f+g)(x) = x^2+x-1$, $(f-g)(x) = x^2-x+1$

$(f \cdot g)(x) = x^3-2x+x-2$, $(f/g)(x) = \frac{x^2+1}{x-2}$

Composition

Composition is applying a function to the output of another function: $(g \circ f)(x) = g(f(x))$

With the black box perspective:

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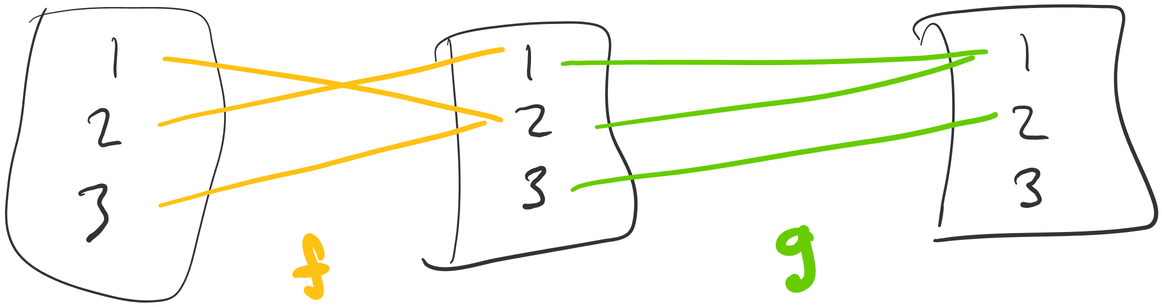
$$x \to \boxed{f} \to \boxed{g} \to g(f(x))$$

careful of order!



→ Revisit Geogebra Visualization

How can we calculate a formula for $g \circ f$?

↳ Plug the formula for f into the formula for g.

e.g. $f(x) = x^2+2$, $g(x) = \frac{1}{x}$

↳ $(g \circ f)(x) = \frac{1}{x^2+2}$

Questions

Q1 How does the domain of $g \circ f$ compare to that of f?

Q2 How does the range of $g \circ f$ compare to that of g?

Q3 Does order matter? $f+g \overset{?}{=} g+f$, $f \cdot g \overset{?}{=} g \cdot f$

$\frac{f}{g} \overset{?}{=} \frac{g}{f}$, $g \circ f \overset{?}{=} f \circ g$

Warning: Don't confuse $g \circ f$ with $g \cdot f$

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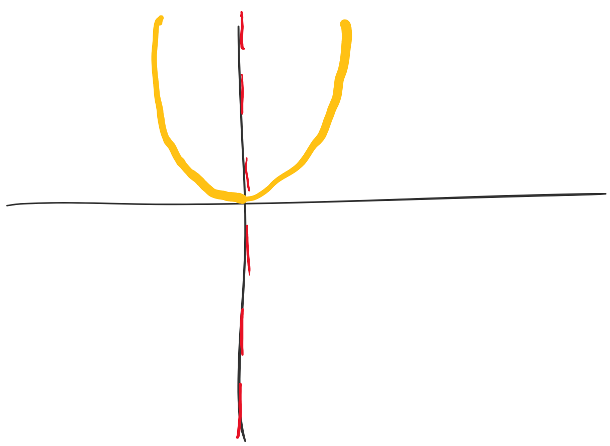
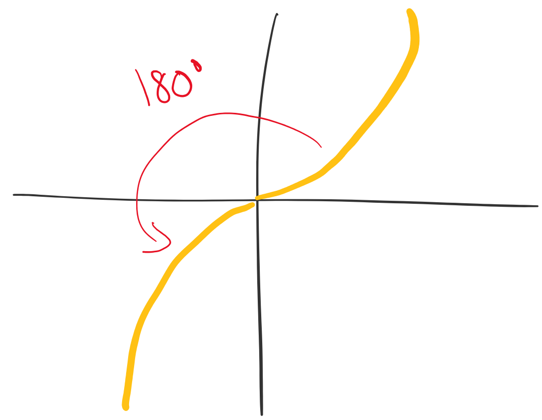
composition vs. product

Symmetry



reflection symmetry about the y-axis



(180°)-rotation symmetry about the origin

Can a graph of a function be symmetric about the x-axis?

Some graphs have symmetry. If the graph of f has reflection symmetry about the y-axis, we call f **even.** If the graph of f has rotation symmetry about the origin, we call f **odd.**

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How can we express evenness and oddness with an equation?

$f(-x) = f(x) \rightarrow f$ is even

$f(-x) = -f(x) \rightarrow f$ is odd

Why do you think these functions are called even/odd?

Piecewise-Defined Functions

These functions are defined by different equations on different parts of their domain.

For example,

$$f(x) = \begin{cases} x+2 & x < -2 \\ 0 & -2 \le x < 0 \\ x^2 & 0 \le x \end{cases}$$

Q: Are p.w. functions always well-defined?
i.e. Can I write a "bad" definition of one?

One special piecewise-defined function is the absolute value

$$|x| = \begin{cases} x & x \ge 0 \\ -x & x \le 0 \end{cases}$$

Intuition: "makes everything positive (non-negative)"
"measures distance"

"measures distance"

What kind of function is $|\cdot|$?