Week 1 - Algebra Review

Linear functions have the form $f(x) = ax + b$

for constants $a, b \in \mathbb{R}$.

The slope of a line through two points $(x_1, y_1), (x_2, y_2)$

is

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

How do we know that the slope of $f(x) = ax + b$ is a?

Pick two points: $(0, b)$, $(1, a+b)$

$$m = \frac{a + b - b}{1 - 0} = \frac{a}{1} = a$$

If a is the slope, what is b? y-intercept

We call $f(x) = mx + b$ the slope-intercept form.

Other Forms

- Point-slope form: $f(x) - y_1 = m(x - x_1)$
  Useful for finding slope-intercept form!
- Standard form: $ax + by = c$

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Warning Functions vs. lines

slope-intercept and point-slope form are both great for describing linear functions, which have graphs that are lines. However, standard form can express vertical lines, which are not functions

Practice

Find the slope-intercept and standard form of the line passing through $(1,-3)$ and $(-3,5)$

$m=\frac{5-(-3)}{-3-1}=\frac{8}{-4}=-2$, $y-(-3)=-2(x-1)$

$y+3=-2x+2$

$y+2x=-1 \leftarrow y=-2x-1$

Polynomials

A polynomial function has the form

$$f(x)=a_nx^n+a_{n-1}x^{n-1}+\ldots+a_1x+a_0$$

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0$$

The degree of a polynomial is the largest n for which a_n is non-zero.

If $f(x)$ is a polynomial of degree...

0	we call it a	constant function
1	we call it a	linear function
2	we call it a	quadratic function
3	we call it a	cubic function

A power function is a polynomial with only one term

e.g. x^2, $3x^5$, $-x$, 5, ...

Behavior at Infinity

What happens to $f(x)$ as we plug in larger and larger numbers for x?

If $f(x) = x$, then $f(x)$ also gets bigger, so we say

$f(x) \to \infty$ as $x \to \infty$, and $f(x) \to -\infty$ as $x \to -\infty$

what about $f(x) = x^2$? $f(x) = \frac{1}{x}$? $f(x) = x^3 + x^2$?

Zeroes of Polynomials

How do we find zeroes of functions? Solve $f(x) = 0$.

Easy for degree 0 and degree 1 polynomials.

What about for degree 2 (quadratic)?

↳ Quadratic Formula $\quad x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

How many solutions does the formula give us?

$b^2 - 4ac$ is called the discriminant

How can we find the zeroes of higher-degree polys?

↳ It's hard. Don't worry about it.