Week 2 - Average Rate of Change

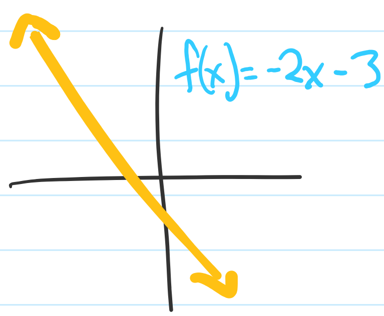
Thursday, September 19, 2019 1:28 PM

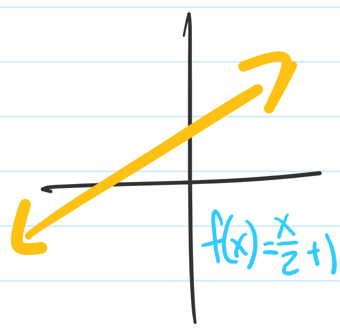
What is the rate of change of a function?

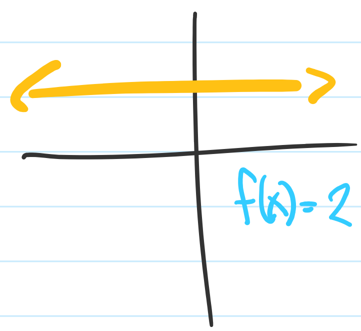
= How $f(x)$ changes when x changes

When the graph $y=f(x)$ is a line, the rate of change is given by the slope of the line.

Remark: Linear functions always have a constant slope.

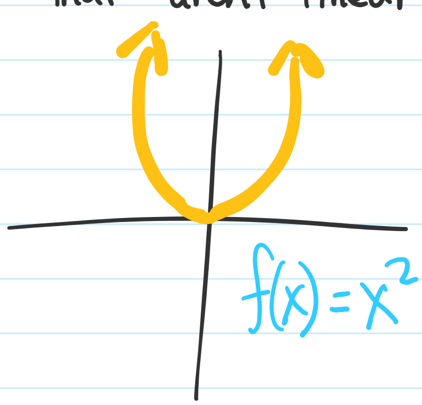


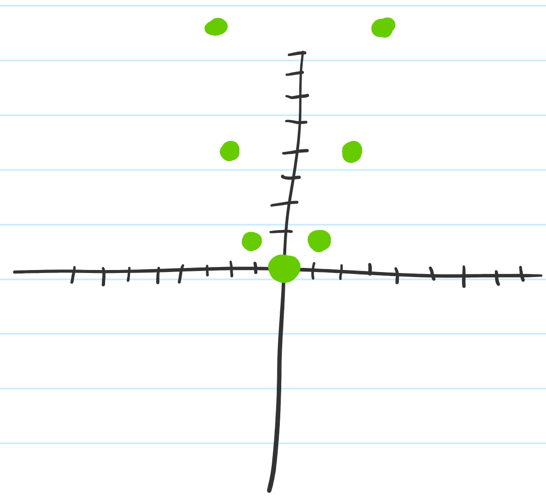




What can we say about the rate of change of functions that aren't linear?

e.g.





$f(0)=0$

$f(1)=1$

$f(2)=4$

> small change

rate of change is

$f(2)=4$
$f(3)=9$ > big change > rate of change is not constant!

Idea: we can approximate the rate of change of a function near some point $(a, f(a))$ by taking another point $(x, f(x))$ on the graph, drawing a line through the points, and calculating the slope.

This kind of line is called a secant line.

The closer x is to a, the better the slope of the secant approximates the rate of change.

The secant lines approach the tangent line to f at a.

The slope of the tangent line measures the rate of change of f at a.

We can define a function

$$f'(a) = (\text{the slope of the tangent line to } f \text{ at } a)$$

called the derivative of f.

Physics

Idea: Velocity = rate of change of position

Average velocity : If $s(t)$ is the position of an object at time t, then the object's average velocity on $[a,b]$ is velocity on $[a,b]$ is

$$v_{avg} = \frac{s(b)-s(a)}{b-a}$$

The instantaneous velocity @ $t=a$ is then defined as the value that the average velocity approaches as $b \to a$.

This idea of "approaching" a value is called a limit.

Practice:

$f(x) = 9-4x^2$

Estimate the rate of change near $a=1$ using the points

$\{-2,-1,0,1,2\}$

x	$f(x)$
-2	-7
-1	5
0	9
1	5
2	-7

-8 $\quad -4 \quad -12 \quad 0 \quad 4$