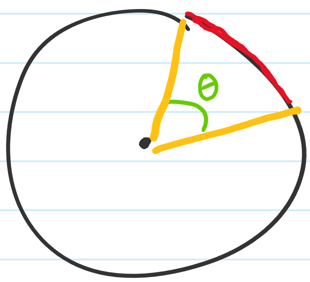
Week 2 - Trig Functions (§1.3)

Wednesday, September 25, 2019 2:04 PM

Radians, like degrees, are a unit for measuring angles.

We can convert between radians and degrees with a unit circle:

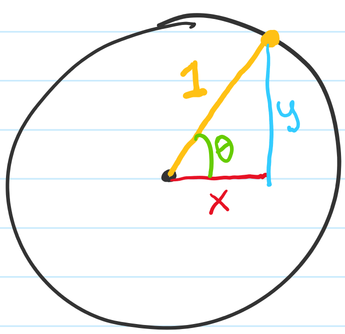


The measure of θ in radians is the length of s.

So $360° = 1$ full circle $= 2\pi r$.

Important values to know

0	30	45	60	90
0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$



We can use the unit circle to define trig functions too:

$\sin\theta = \frac{y}{1} = y$ $\qquad$ $\csc\theta = \frac{1}{y}$

$\cos\theta = \frac{x}{1} = x$ $\qquad$ $\sec\theta = \frac{1}{x}$

$\tan\theta = \frac{y}{x}$ $\qquad$ $\cot\theta = \frac{x}{y}$

If we let the circle's radius r vary, we can write any point (x, y) in terms of (r, θ) and vice versa. These are called polar coordinates.

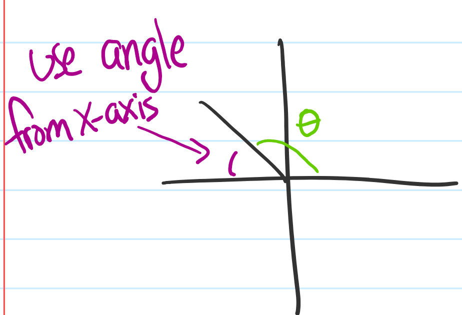
$x = r\cos\theta$

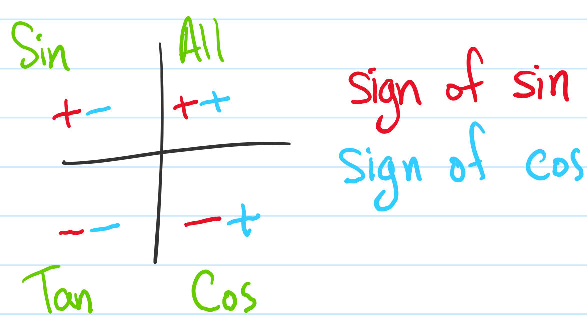
$$\begin{cases} x = r\cos\theta \\ y = r\sin\theta \end{cases}$$

Again, some important values to know:

θ	$\sin\theta$	$\cos\theta$
0	0	1
$\pi/6$	$1/2$	$\sqrt{3}/2$
$\pi/4$	$\sqrt{2}/2$	$\sqrt{2}/2$
$\pi/3$	$\sqrt{3}/2$	$1/2$
$\pi/2$	1	0

Shortcut: If you know the values of sin, cos on $[0, \pi/2]$, you can figure them out for any angle.





Trigonometric Identities

Book def: "An equation involving trig functions that is true for all angles."

Note: I think they're called identities when they're of the form

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$$(\text{stuff}) = 1$$

Reciprocal Identities:

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cot\theta = \frac{\cos\theta}{\sin\theta}$$

$$\csc\theta = \frac{1}{\sin\theta} \qquad \sec\theta = \frac{1}{\cos\theta}$$

Pythagorean Identities:

$$\sin^2\theta + \cos^2\theta = 1 \qquad 1 + \cot^2\theta = \csc^2\theta$$

$$1 + \tan^2\theta = \sec^2\theta$$

Addition/Subtraction

$$\sin(\alpha \pm \beta) = \sin(\alpha)\cos(\beta) \pm \cos(\alpha)\sin(\beta)$$

$$\cos(\alpha \pm \beta) = \cos(\alpha)\cos(\beta) \mp \sin(\alpha)\sin(\beta)$$

Double-angle Formulas

$$\sin(2\theta) = 2\sin\theta\cos\theta$$

$$\cos(2\theta) = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta = \cos^2\theta - \sin^2\theta$$

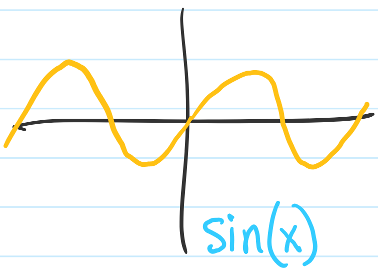
Practice

Solve
- $1 + \cos(2\theta) = \cos\theta$
- $\sin(2\theta) = \tan\theta$

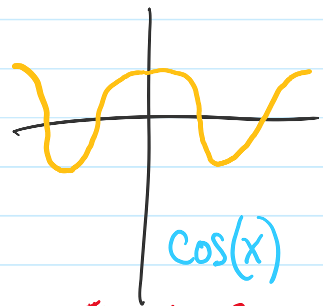
Graphing Trig Functions

Since θ is the same angle as $\theta + 2\pi$, trig functions are periodic.

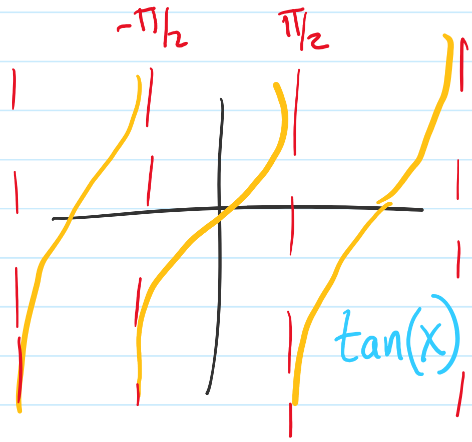
In notation, the period of a function f is the smallest $p > 0$ such that $f(x) = f(x+p)$.



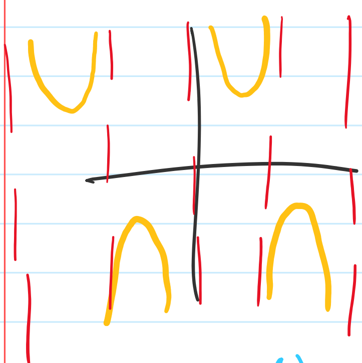
sin(x)



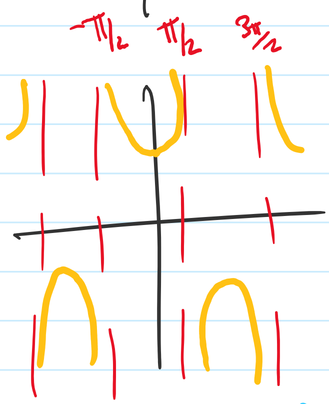
cos(x)



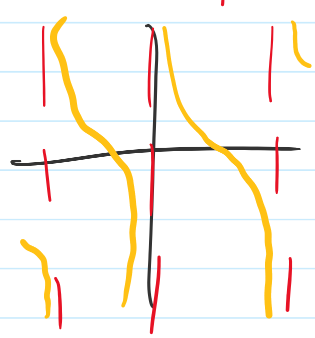
tan(x)



csc(x)



sec(x)



cot(x)

We have special names for properties of transformed trig functions:

If $f(x) = A\sin(B(x - \alpha)) + C$

A is called the amplitude.

B changes the period to $\frac{2\pi}{|B|}$

C causes a vertical shift

B changes the period to $\frac{}{|B|}$

C causes a vertical shift

α causes a "phase shift"