Week 3 - Exponential and Logarithmic Functions (§1.5)

Monday, September 30, 2019 2:24 PM

Properties of powers:

$b^x = \underbrace{b \cdot b \cdot \cdots \cdot b}_{x \text{ times}}$ if x is a positive integer

$b^0 = 1$, $b^{-x} = \frac{1}{b^x}$, $b^{p/q} = \sqrt[q]{b^p}$

Exponential functions are of the form

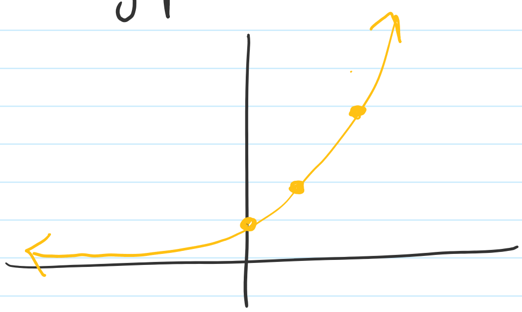
$f(x) = b^x$ for $b > 0$, $b \neq 1$

b: base

x: exponent

Not to be confused w/ power functions $f(x) = x^b$

Their graphs look like



x	2^x
-2	$1/4$
-1	$1/2$
0	1
1	2
2	4

behavior @ $\pm\infty$?

asymptote?

what if $b < 1$?

More exponent rules:

$b^x \cdot b^y = b^{x+y}$ $(b^x)^y = b^{xy}$ $(ab)^x = a^x b^x$

$\frac{b^x}{b^y} = b^{x-y}$ $\frac{a^x}{b^x} = \left(\frac{a}{b}\right)^x$

Practice

Simplify:

1) $\frac{8^{-1/3}}{2^{-1}} = \frac{2}{8^{1/3}} = \frac{2}{2} = 1$

2) $(27x)^{1/3}$ $27^{1/3} x^{1/3} x^2$ $3 \; {}^{7/3}$

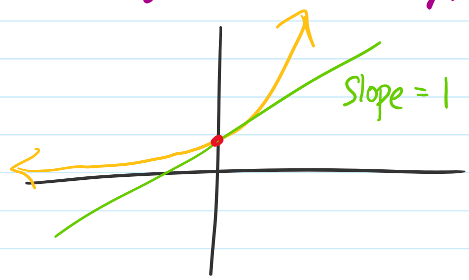
2) $\dfrac{(27x)^{1/3}}{2x^{-2}} = \dfrac{27^{1/3}\,x^{1/3}\,x^{2}}{2} = \dfrac{3}{2}x^{7/3}$

$e = 2.718282\ldots$ is a special real number. called Euler's number or the natural base.

It arises as the limit of $\left(1+\frac{1}{n}\right)^n$ as $n \to \infty$,

but perhaps the more interesting/relevant property is that the slope of the tangent line to the graph of e^x is 1 at $x=0$,

(and is e^x at any x-value, actually).



e^x is called the natural exponential function.

Does $f(x) = 2^x$ have an inverse?

↳ It passes the HLT ✓ So yes.

$$f^{-1}(x) = \log_2 x.$$

In general, $f(x) = b^x$ has an inverse, denoted $\log_b x$.

By properties of inverse functions,

$f(x) = y$ means $f^{-1}(y) = x$

$b^x = y$ means $\log_b y = x$ "what power of b is y?"

For example,

- $\log_2 8 = 3$ because $2^3 = 8$.
- $\log_{10} \frac{1}{100} = -2$ because $10^{-2} = \frac{1}{100}$
- $\log_b 1 = 0$ because $b^0 = 1$

also means

$\log_b b^x = x$

$b^{\log_b x} = x$

- $\log_{10} 100 = 2$ because $10^2 = 100$
- $\log_b 1 = 0$ because $b^0 = 1$

$$b^{\log_b x} = x$$

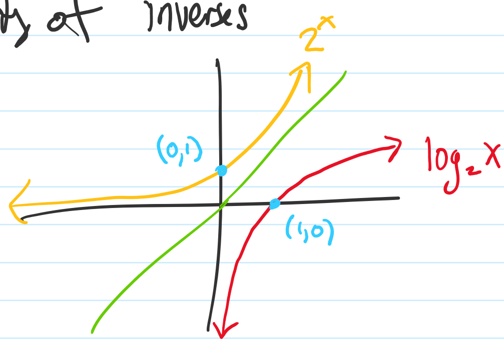
$\log_e x$ has a special name, the natural logarithm, often denoted $\ln(x)$.

e.g. $\ln(1) = 0$, $\ln(e) = 1$, $\ln(e^2) = 2$, ...

Graphs

We can plot points to graph $f(x) = \log_2 x$, or use the reflection property of inverses

asymptote?



More properties of logs

$$\log_b(ac) = \log_b(a) + \log_b(c)$$

$$\log_b\left(\frac{a}{c}\right) = \log_b(a) - \log_b(c)$$

$$\log_b(a^r) = r \log_b(a)$$

Practice

Solve for x:

1) $2^x = 9$ $\boxed{x = \log_2 9}$
2) $e^x - 2e^{-x} = 1$ $\quad e^{2x} - e^x - 2 = 0 \rightsquigarrow (e^x - 2)(e^x + 1)$

$e^x = 2$ or $e^x = -1$

$\boxed{x = \ln 2}$

Alternate solution to #1

$\ln(2^x) = \ln(9) \rightsquigarrow x\ln(2) = \ln(9) \rightsquigarrow x = \frac{\ln 9}{\ln 2}$

Alternate solution to #1

$\ln(2^x) = \ln(9) \rightsquigarrow x\ln(2) = \ln(9) \rightsquigarrow x = \frac{\ln 9}{\ln 2}$

Change of base

$\log_a x = \frac{\log_b x}{\log_b a}$ $\left(\text{if } b=e, \quad \log_a x = \frac{\ln x}{\ln a}\right)$

also:

$a^x = b^{x\log_b a}$ $\left(\text{if } b=e, \ a^x = e^{x\ln a}\right)$

(No hyperbolic functions)

End of potential midterm #1 material