Week 3 - Sequences

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A sequence is a list of numbers like

1, 3, 5, 7, 9, ... ← if it ends, it's a finite sequence
if not, it's an infinite sequence

terms

If we want to write a generic sequence in variables:

$a_1, a_2, a_3, a_4, \ldots = \{a_n\}_{n=1}^{\infty}$, also written $\{a_n\}$

n is called the index.

We can write explicit formulas for the n^{th} term in a sequence like

$a_n = 2n-1$ or $\{2n-1\}_{n=1}^{\infty}$ ← Same sequence

Or recursively as $\underbrace{a_1 = 1, \ a_n = a_{n-1} + 2}$

called a recurrence relation

How can we write 2, 4, 8, 16, 32, ... in 3 different ways?

$a_n = 2^n$, $\{2^n\}_{n=1}^{\infty}$, $a_1 = 2, \ a_n = 2a_{n-1}$

Types of Sequences

In an arithmetic sequence,

the difference between consecutive terms is the same.

e.g. 5, 8, 11, 14, ... $\boxed{a_n = 3n+2}$ in general, e.g. $5, 8, 11, 14, \ldots$ $\boxed{a_n = 3n+2}$

$2, 1, 0, -1, -2 \ \ldots$ $\boxed{a_n = 3-n}$

in general, $\boxed{a_n = cn + b}$

What recurrence relations describe the above?

What explicit formulas describe the above?

In a geometric sequence,

the ratio of consecutive terms is the same.

e.g. $3, 9, 27, 81, \ldots$ $\boxed{a_n = 3^n}$

$5, 10, 20, 40, 80, \ldots$ $\boxed{a_n = 5 \cdot 2^{n-1}}$

in general, $\boxed{a_n = cr^n}$

Recurrence relations? Explicit formulas?

Practice

Find an explicit formula for each sequence and say if it is arithmetic and/or geometric.

1) $3, -1, \frac{1}{3}, -\frac{1}{9}, \frac{1}{27}, \ldots$ $a_n = 3 \cdot \left(-\frac{1}{3}\right)^{n-1}$ geo

2) $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}, \ldots$ $a_n = \frac{n}{n+1}$ neither

3) $-2, -2, -2, -2, -2, \ldots$ $a_n = -2$ both
