Week 3 - The Monotone Convergence Theorem

Tuesday, October 8, 2019 1:38 PM

A sequence is bounded above if, for some real # M,

$$a_n \le M \quad \text{for all } n.$$

A sequence is bounded below if, for some real # M,

$$a_n \ge M \quad \text{for all } n.$$

If a sequence is both bounded above and below, we just call it bounded.

Else, we call it unbounded.

e.g. $\{\frac{1}{n}\}$ is bounded: $0 \le \frac{1}{n} \le 1$

Is $\{2^n\}$ bounded (above/below)? → bounded below, not above.

Theorem: If a sequence converges, then it is bounded.

Is the converse true? → No, e.g. $\sin(n)$ or $(-1)^n$.

A sequence is increasing if $a_n \le a_{n+1}$ for all n.

A sequence is decreasing if $a_n \ge a_{n+1}$ for all n.

A sequence is monotone if it is increasing or decreasing.

Some sequences are increasing / decreasing / monotone if we ignore the first few terms. These are called eventually increasing / decreasing / monotone.

The Monotone Convergence Theorem

The Monotone Convergence Theorem

If a_n is bounded and eventually monotone, then a_n is convergent.

Ex. $a_n = \left\{\frac{1}{2^n}\right\}$

bounded : $0 \le \frac{1}{2^n} \le 1$ ✓

eventually monotone? $\frac{1}{2^n} \ge \frac{1}{2^{n+1}}$ ✓

↳ must be convergent!

$\lim_{n\to\infty} a_n = \lim_{n\to\infty} 2a_{n-1} = L$, so $L=2$