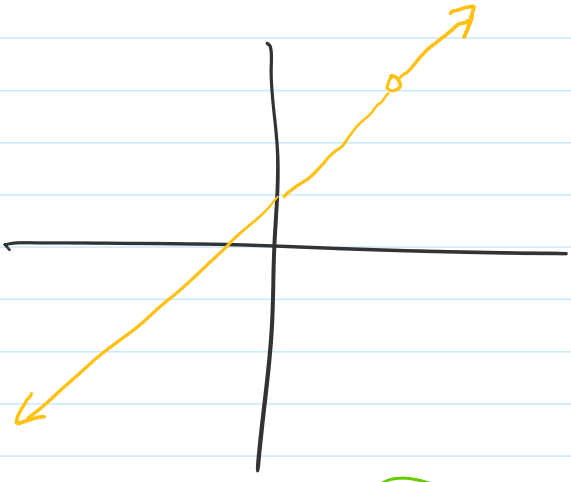
Week 4 - The Limit of a Function

Tuesday, October 8, 2019 2:03 PM

Consider the graph of $f(x) = \frac{x^2-4}{x-2}$.

The function is not defined at $x=2$, but nearby the graph appears to approach the point $(2,4)$.

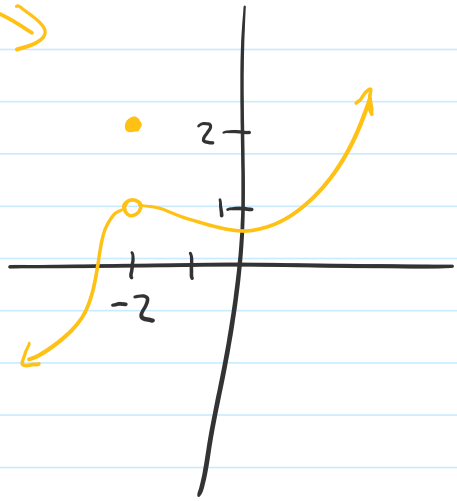


We say that $\lim_{x \to 2} f(x) = 4$.

Intuitively, the limit of $f(x)$ at a is, if it exists, a number L such that $f(x)$ gets closer to L as x gets closer to a.

We write: $\lim_{x \to a} f(x) = L$.

Warning! On the graph of $f(x)$



$\lim_{x \to -2} f(x) = 1$, even though $f(-2) = 2$.

Also, limits may not exist!

$f(x) = \sin\left(\frac{1}{x}\right)$ has no limit as $x \to 0$.

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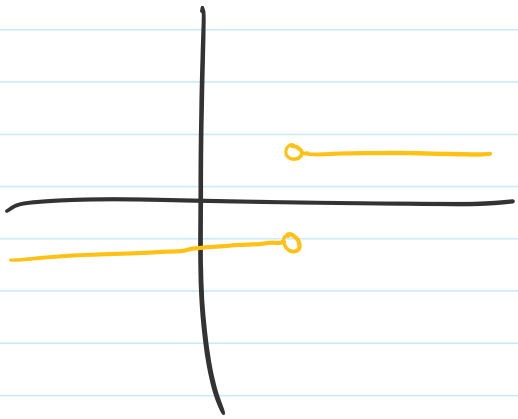
One-Sided Limits

Consider the graph of $f(x) = \frac{|x-2|}{x-2}$

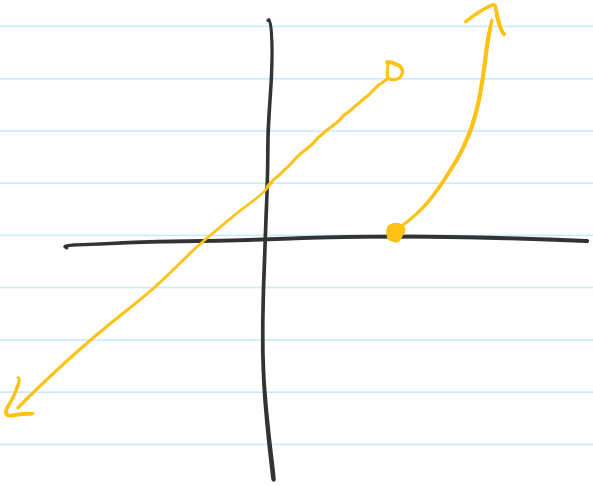


Approaching $x=2$ from the left, it looks like $f(x) \to -1$, but from the right, $f(x) \to 1$. We say

$\lim_{x \to 2^-} f(x) = -1$ and $\lim_{x \to 2^+} f(x) = 1$

these are called one-sided limits.

e.g. $f(x) = \begin{cases} x+1 & x < 2 \\ x^2 - 4 & x \geq 2 \end{cases}$



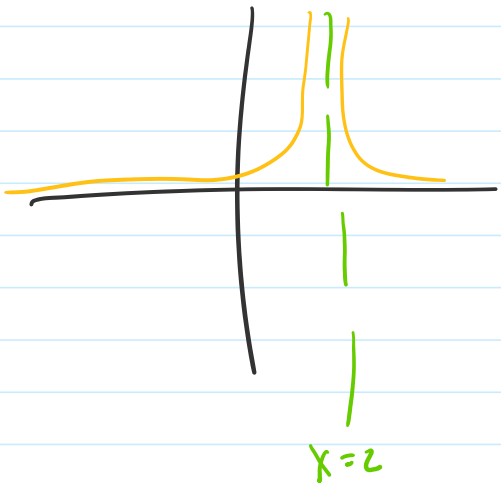
What is $\lim_{x \to 2^-} f(x) = 3$

$\lim_{x \to 2^+} f(x) = 0$

Fact:

$\lim_{x \to a} f(x) = L$ if and only if $\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x) = L$

Consider the graph of $f(x) = \frac{1}{(x-2)^2}$



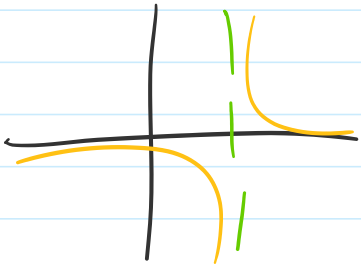
We say that

$\lim_{x \to 2} f(x) = \infty$

This is an example of an infinite limit.

Infinite limits can be one-sided as well:

For $g(x) = \frac{1}{x-2}$,



$\lim_{x \to 2^-} g(x) = -\infty$, but $\lim_{x \to 2^+} g(x) = \infty$

If $\lim_{x \to a^\pm} f(x) = \pm\infty$, we call the line $x=a$ a

vertical asymptote of f.

Practice

Use the graph to find the limits:

1) $\lim_{x \to -1} f(x) = -1$

2) $\lim_{x \to 1} f(x) =$ DNE

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3) $\lim_{x \to 0} f(x) = 0$

4) $\lim_{x \to -2} f(x) =$ DNE

5) $\lim_{x \to 1^-} f(x) = -\infty$