

Week 5 - Limit Laws

Tuesday, October 8, 2019 3:40 PM

Earlier, we talked about rules that help us find limits of sequences.
Now, we have rules to help us find limits of functions.

Limit Laws $\left(\lim_{x \rightarrow a} f(x) = L, \lim_{x \rightarrow a} g(x) = M \right)$

1) $\lim_{x \rightarrow a} c = c$

2) $\lim_{x \rightarrow a} x = a$

3) $\lim_{x \rightarrow a} f(x) \pm g(x) = \lim_{x \rightarrow a} f(x) \pm \lim_{x \rightarrow a} g(x) = L \pm M$ "sum/difference"

4) $\lim_{x \rightarrow a} f(x) \cdot g(x) = \lim_{x \rightarrow a} f(x) \cdot \lim_{x \rightarrow a} g(x) = L \cdot M$ "product"

5) $\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \frac{\lim_{x \rightarrow a} f(x)}{\lim_{x \rightarrow a} g(x)} = \frac{L}{M}$, $M \neq 0$ "quotient"

6) $\lim_{x \rightarrow a} f(x)^n = \left(\lim_{x \rightarrow a} f(x) \right)^n = L^n$, $L \geq 0$ if n even "power/root"

Ex.

$$\lim_{x \rightarrow 1} (4x + 5) = \lim_{x \rightarrow 1} 4x + \lim_{x \rightarrow 1} 5 = 4 \lim_{x \rightarrow 1} x + \lim_{x \rightarrow 1} 5 = 4(1) + 5 = 9$$

Fact: If $f(a)$ exists, then $\lim_{x \rightarrow a} f(x) = f(a)$ as long as f is a rational function.

f is a rational function.

What if $f(a)$ doesn't exist?

More Limit Techniques

If $\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \frac{0}{0}$, we can try cancelling factors from the numerator and denominator.

$$\text{e.g. 1) } \lim_{x \rightarrow 1} \frac{x^2 - 1}{x - 1} = \lim_{x \rightarrow 1} \frac{(x+1)\cancel{(x-1)}}{\cancel{(x-1)}} = \lim_{x \rightarrow 1} x+1 = 2.$$

$$\begin{aligned} 2) \lim_{x \rightarrow -1} \frac{\sqrt{x+2} - 1}{x+1} &= \lim_{x \rightarrow -1} \frac{x+2-1}{(x+1)(\sqrt{x+2}+1)} = \frac{\cancel{x+1}}{(x+1)(\sqrt{x+2}+1)} \\ &= \lim_{x \rightarrow -1} \frac{1}{\sqrt{x+2}+1} = \frac{1}{\sqrt{1}+1} = \frac{1}{2}. \end{aligned}$$

Practice

Evaluate: 1) $\lim_{x \rightarrow 0} \left(\frac{1}{x} + \frac{5}{x(x-5)} \right)$

Hint: Combine the terms first

2) $\lim_{x \rightarrow 5} \frac{\sqrt{x-1} - 2}{x-5}$

3) $\lim_{x \rightarrow 5} \sqrt{x-5}$

4) $\lim_{x \rightarrow 5} \sqrt{x-5}$

$$4) \lim_{x \rightarrow 5^+} \sqrt{x-5}$$

