Week 5 - Continuity

Monday, October 14, 2019 12:20 PM

$f(x)$ is continuous at a if:

1) $f(a)$ is defined

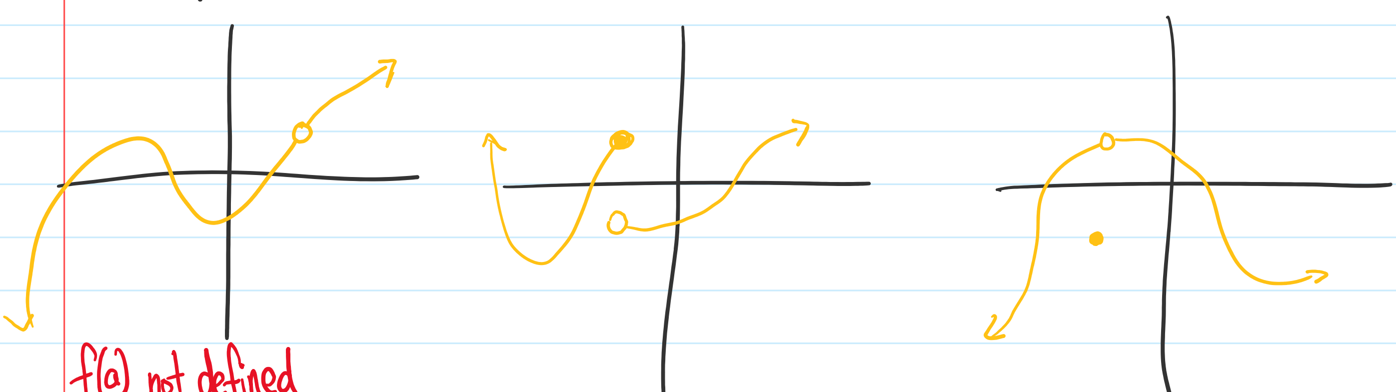
2) $\lim\limits_{x \to a} f(x)$ exists

3) $\lim\limits_{x \to a} f(x) = f(a)$

Idea: You can draw the graph of a continuous function without lifting your pen.

Otherwise, we say $f(x)$ is discontinuous at a.

Examples:



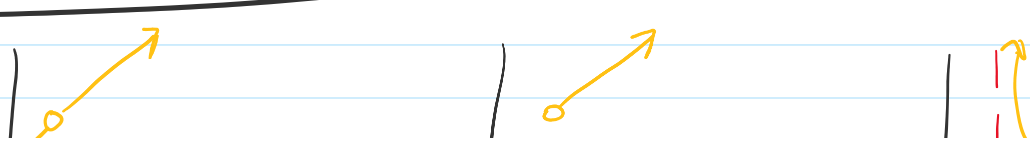
$f(a)$ not defined

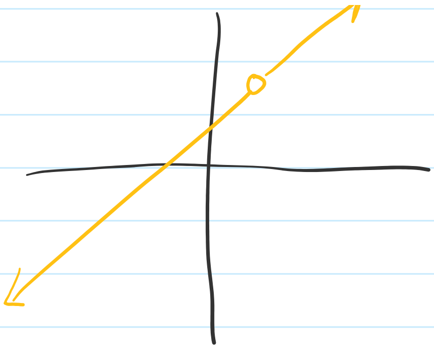
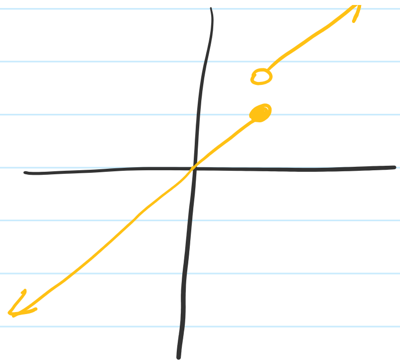
$\lim\limits_{x \to a} f(x)$ DNE

$f(a) \neq \lim\limits_{x \to a} f(x)$

Fact: Polynomials and rational functions are continuous at every point in their domain.

Types of Discontinuities



		
$f(x) = \dfrac{x^2-4}{x-2}$	$g(x) = \begin{cases} x+1 & x>2 \\ x & x \le 2 \end{cases}$	$h(x) = \dfrac{1}{x-2}$
$\boxed{\text{removable discontinuity}}$	$\boxed{\text{jump discontinuity}}$	$\boxed{\text{infinite discontinuity}}$
"hole in the graph"	"parts of function not connected"	"vertical asymptote"
$\lim\limits_{x\to a} f(x)$ exists	$\lim\limits_{x\to a^-} f(x) \neq \lim\limits_{x\to a^+} f(x)$ (neither is $\pm\infty$)	$\lim\limits_{x\to a^-} f(x)$ or $\lim\limits_{x\to a^+} f(x) = \pm\infty$
cancelling terms in fractions	piecewise stuff	when you can't cancel stuff in fractions

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eg. Check if the function is continuous @ the given point

$$f(x) = \begin{cases} x^2+2 & x<3 \\ x+5 & x \ge 3 \end{cases} \quad @\ x=3$$

$$g(x) = \begin{cases} -x+1 & x<0 \\ x+1 & x \ge 0 \end{cases} \quad @\ x=0$$

$f(x)$ is continuous from the right at a if $\lim\limits_{x\to a^+} f(x) = f(a)$

$f(x)$ is continuous from the left at a if $\lim\limits_{x\to a^-} f(x) = f(a)$

$f(x)$ is continuous from the left at a if $\lim_{x \to a^-} f(x) = f(a)$

$f(x)$ is continuous over an interval $[a, b]$ if

- $f(x)$ is right-continuous at a
- $f(x)$ is continuous at x while $a < x < b$
- $f(x)$ is left-continuous at b

e.g. $f(x) = \sqrt{x}$ is continuous on $[0, \infty)$

$g(x) = \frac{1}{\sqrt{x}}$ is continuous on $(0, \infty)$

The Composite Function Theorem

If $f(x)$ is continuous at L and $\lim_{x \to a} g(x) = L$, then

$$\lim_{x \to a} (f \circ g)(x) = \lim_{x \to a} f(g(x)) = f\left(\lim_{x \to a} g(x)\right) = f(L).$$

Fact: All trig functions are continuous at all pts. in their domain.

e.g. 1) $\lim_{x \to \pi} \sin\left(\frac{x}{2}\right) = \sin\left(\frac{\pi}{2}\right) = 1.$

2) $\lim_{x \to 0^+} \tan\left(\frac{x}{\ln x}\right) = \tan(0) = 0$

Practice

1) Classify all discontinuities:

a) $f(x) = \frac{1}{\ldots}$ $\quad$ $x = 3$: infinite

Classify all discontinuities

a) $f(x) = \frac{1}{x^2+4x+4}$ $\quad x=-2$: infinite

b) $g(x) = \frac{x}{|x|}$ $\quad x=0$: jump

c) $h(x) = \sin(x)\csc(x)$ $\quad x = 0, \pi, 2\pi, 3\pi, \ldots$: removable