Week 6 - The Derivative (§3.1)

Thursday, October 17, 2019 1:41 PM

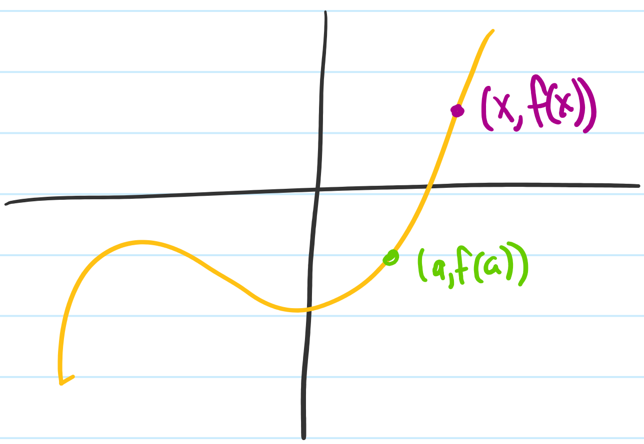
Now that we know how to find limits, we can finally compute the slope of the tangent line!

Recall the slope formula:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

If we want to find the slope of the secant line between some point $(a, f(a))$ and $(x, f(x))$

$$m = \frac{f(x) - f(a)}{x - a}$$



This quantity is called a "difference quotient"

The slope of the secant line, m, approaches the slope of the tangent line at a when x is close to a. Therefore:

The slope of the tangent line to f(x) at a is:

$$\lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

If we let $x = a+h$, then

$$m = \frac{f(a+h) - f(a)}{a+h-a} = \frac{f(a+h)-f(a)}{h}$$

Alternatively, the slope of the tangent line to $f(x)$ at a is:

$$\lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

This value is called the derivative of $f(x)$ at a.

Written:

$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x-a} = \lim_{h \to 0} \frac{f(a+h)-f(a)}{h}$$

"f- prime of a"

e.g.

$f(x) = 3x$, $a = 5$ $\rightsquigarrow f'(5) = \lim_{h \to 0} \frac{3(5+h) - 3(5)}{h} = \lim_{h \to 0} \frac{15 + 3h - 15}{h} = 3$

$g(x) = x^2 + x$, $a = 2$ $\rightsquigarrow g'(2) = \lim_{h \to 0} \frac{f(2+h) - f(2)}{h} = \frac{(2+h)^2 + (2+h) - (2^2+2)}{h}$

$= \frac{2^2 + 4h + h^2 + 2 + h - 6}{h} = \frac{5h + h^2}{h} = 5 + h = 5$

$h(x) = |x|$, $a = -1$ $\rightsquigarrow h'(-1) = \lim_{h \to 0} \frac{|-1+h| - |-1|}{h} = \frac{1 - h - 1}{h} = \frac{-h}{h} = -1$

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slopes of secant lines $\rightsquigarrow$ average rate of change of $s(t)$
(average velocity of object)

slope of tangent line $\rightsquigarrow$ instantaneous rate of change of $s(t)$
(instantaneous velocity of object)

So, we can find the instantaneous velocity at time a by finding $s'(a)$.

e.g.

$$s(t) = 10 - 5t^2 \ (m/s), \quad t = 1 \ (s)$$

$$s'(1) = \lim_{h \to 0} \frac{s(h+1) - s(1)}{h} = \frac{10 - 5(h+1)^2 - (10-5)}{h}$$

$$= \frac{10 - 5(h^2 + 2h + 1) - 5}{h} = \frac{10 - 5h^2 - 10h - 5 - 5}{h}$$

$$= \frac{-5h^2 - 10h}{h} = -5h - 10 = \boxed{-10.} \ (m/s)$$

Practice

Compute:

1) $f'(3)$, if $f(x) = x^2 + 1$

2) $g'(2)$, if $g(x) = \frac{1}{x}$