Week 6 - The Derivative Function

Tuesday, October 22, 2019 3:15 PM

Recall: The derivative of $f(x)$ at a is given by

$$f'(a) = \lim_{x\to a} \frac{f(x)-f(a)}{x-a} = \lim_{h\to 0} \frac{f(a+h)-f(a)}{h}$$

We're going to use this definition more today.

e.g. $f(x) = x^2$, $a=1$ or $a=2$

The Derivative Function

Let $f(x)$ be a function. We can define the derivative function $f'(x)$ as:

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

e.g. $f(x) = x^2$; $f'(x) = 2x$

f is differentiable at a if $f'(a)$ exists.

Q: Give an example of a function not differentiable at a point.

If f is differentiable at every point in its domain, we call f a differentiable function.

eg. $f(x) = \sqrt{x}$, $f'(x) = \frac{1}{2\sqrt{x}}$

The derivative of $y = f(x)$ is written many different ways:

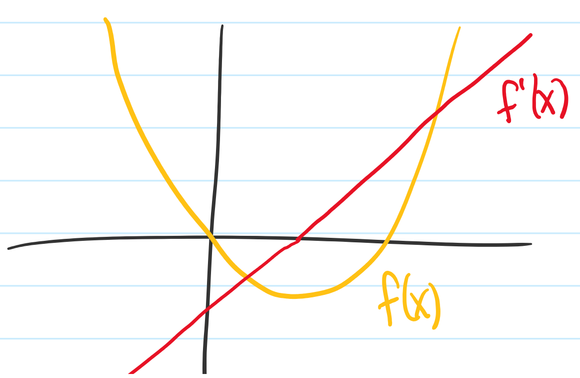
Lagrange $\rightarrow$ $f'(x)$, $\frac{d}{dx}(f(x))$, y', Leibniz $\rightarrow$ $\frac{dy}{dx}$, $\frac{df}{dx}$, $\dot{y}$ $\leftarrow$ Newton, ...

Specifically, $\frac{dy}{dx}$ is called Leibniz notation, and is motivated by the observation that the slopes of secant lines look like

$$m = \frac{\Delta y}{\Delta x}, \quad \text{so} \quad \text{"} \frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} \text{"}$$

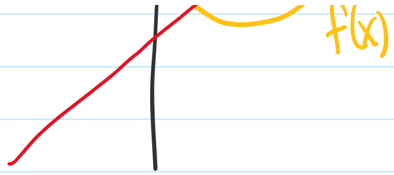
Graphing the Derivative

$f(x) = x^2 - 2x$, $f'(x) = 2x - 2$



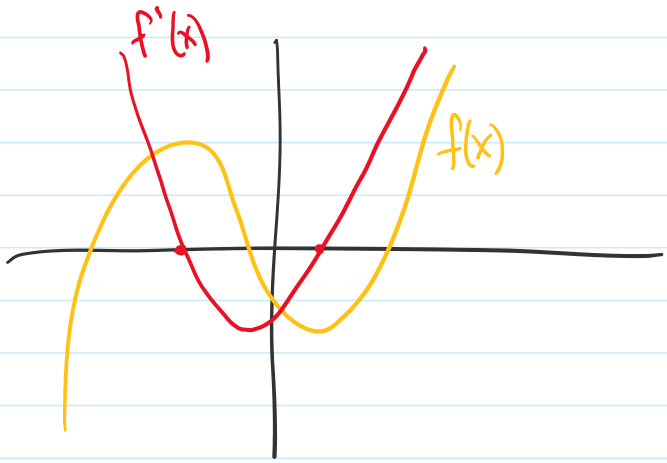
Note that $f'(x)$ is

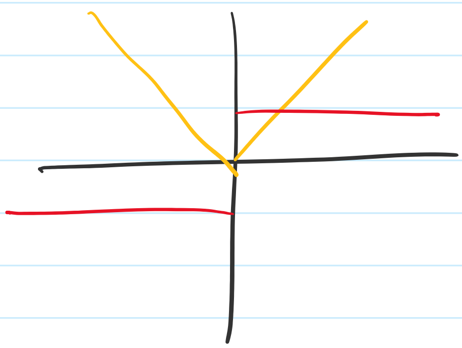
- pos. when $f(x)$ increasing
- neg. when $f(x)$ decreasing.



neg. ⇒ f(x) decreasing.

Let's try sketching $f'(x)$ based on $f(x)$:





We can take derivatives of derivatives, and derivatives of derivatives of derivatives, etc.

These are called higher-order derivatives.

Denoted $f''(x)$, $f'''(x)$, ...

$\frac{d^2y}{dx^2}$, $\frac{d^3y}{dx^3}$, ...

$\ddot{y}$, $\dddot{y}$, ...

$\ddot{y}$, $\dddot{y}$, ...

e.g. $f(x) = x^2$, $f'(x) = 2x$, $f''(x) = 2$