Week 6 - More Derivatives

Wednesday, October 23, 2019 1:58 PM

Recall that a function $f(x)$ is continuous at a if

1) $f(a)$ is defined
2) $\lim_{x\to a} f(x)$ exists
3) $\lim_{x\to a} f(x) = f(a)$

$f(x)$ is differentiable at a if

$$f'(a) = \lim_{h\to 0} \frac{f(a+h)-f(a)}{h} \quad \text{exists.}$$

Theorem: If $f(x)$ is differentiable at a, then $f(x)$ is also continuous at a.

Q: Are there continuous functions that aren't differentiable?

Try $f(x) = \sqrt[3]{x}$:

$$f'(0) = \lim_{h\to 0} \frac{\sqrt[3]{0+h} - \sqrt[3]{0}}{h} = \frac{\sqrt[3]{h}}{h} = \frac{1}{h^{2/3}} = \boxed{+\infty}$$

So $f(x)$ not differentiable at $x = 0$.

(Also, piecewise functions like $|x|$.)

Exercise: Find the value of c that makes $f(x)$ differentiable:

$f(x) = \begin{cases} x^2+2 & x < 1 \end{cases}$

$$f(x) = \begin{cases} x^2+2 & x \le 1 \\ c(x-1)+3 & x \ge 1 \end{cases}$$

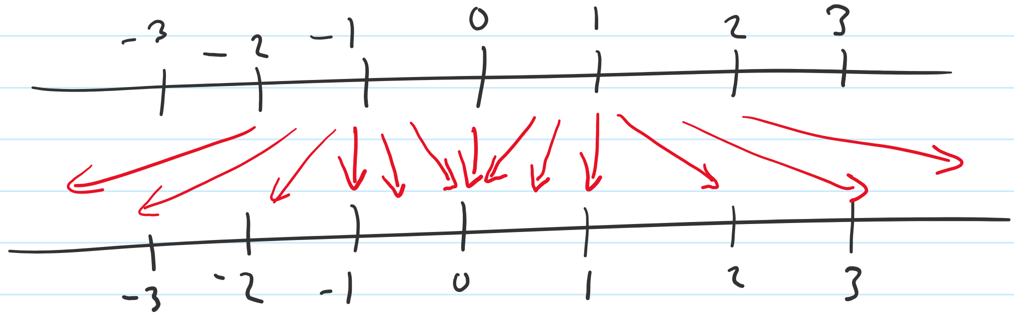
what is $f'(x)$? Is $f'(x)$ differentiable?

Practice

1) Find $f'(x)$ when $f(x) = \sqrt{x}$

Another way to view the derivative is that it measures how much the function "stretches" or "compresses" points on the real number line.

$f(x) = x^3$



↳ See visualization on the website under Links.
