Week 7 - Differentiation Rules

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The Constant Rule: c

$$\frac{d}{dx}(c) = \lim_{h\to 0} \frac{c-c}{h} = \lim_{h\to 0} \frac{0}{h} = \lim_{h\to 0} 0 = \boxed{0}$$

So $\frac{d}{dx}(c) = 0$

The Power Rule: x^n

$$\frac{d}{dx}(x^n) = \lim_{h\to 0} \frac{(x+h)^n - x^n}{h} = \lim_{h\to 0} \frac{x^n + nhx^{n-1} + h^2(\ldots) - x^n}{h}$$

$$= \lim_{h\to 0} nx^{n-1} + h(\ldots) = \boxed{nx^{n-1}}$$

So $\frac{d}{dx}(x^n) = nx^{n-1}$

The Sum/Difference Rule: $f(x) + g(x)$

$$\frac{d}{dx}(f(x)+g(x)) = \lim_{h\to 0} \frac{f(x+h)+g(x+h)-f(x)-g(x)}{h}$$

$$= \lim_{h\to 0} \frac{f(x+h)-f(x)+g(x+h)-g(x)}{h}$$

$$= \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} + \lim_{h\to 0} \frac{g(x+h)-g(x)}{h}$$

$$= \boxed{f'(x) + g'(x)}$$

So $\frac{d}{dx}\left(f(x)+g(x)\right) = \frac{d}{dx}\left(f(x)\right) + \frac{d}{dx}\left(g(x)\right)$

The Constant Multiple Rule : $c f(x)$

$$\frac{d}{dx}\left(cf(x)\right) = \lim_{h\to 0} \frac{cf(x+h) - cf(x)}{h} = \lim_{h\to 0} c\left(\frac{f(x+h)-f(x)}{h}\right)$$

$$= c \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \boxed{cf'(x)}$$

So $\frac{d}{dx}\left(cf(x)\right) = c\,\frac{d}{dx}\left(f(x)\right)$

Tangent Lines

If we want to find the equation of the tangent line to $f(x)$ at a:

- The slope $m = f'(a)$
- It passes through the point $(a, f(a))$

So we can use point-slope form:

$$y - y_0 = m(x - x_0)$$

$$y - f(a) = f'(a)(x-a) \rightarrow y = f'(a)(x-a) + f(a)$$

Example

Find the equation of the tangent line to

$f(x) = x^3 - x$ at 1

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$f'(x) = 3x^2 - 1$ (power and difference rules)

$f'(1) = 3(1)^2 - 1 = 3 - 1 = 2$, $f(1) = 1^3 - 1 = 0$

So $y = 2(x-1) + 0 = \underline{2x - 2}$ is the equation of the tangent line!