Week 7 - More Differentiation Rules (§3.3)

Monday, October 28, 2019 9:34 AM

The Product Rule: $j(x) = f(x) \cdot g(x)$

$$j'(x) = \lim_{h \to 0} \frac{f(x+h)g(x+h) - f(x)g(x)}{h}$$

$$= \lim_{h \to 0} \frac{f(x+h)g(x+h) - f(x)g(x+h) + f(x)g(x+h) - f(x)g(x)}{h}$$

$$= \lim_{h \to 0} \frac{f(x+h)g(x+h) - f(x)g(x+h)}{h} + \lim_{h \to 0} \frac{f(x)g(x+h) - f(x)g(x)}{h}$$

$$= \lim_{h \to 0} g(x+h) \cdot \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} + \lim_{h \to 0} f(x) \cdot \lim_{h \to 0} \frac{g(x+h) - g(x)}{h}$$

$$= g(x) \cdot f'(x) + f(x) \cdot g'(x)$$

$$= f'(x)g(x) + f(x)g'(x)$$

e.g. $f(x) = x^2 + 1$, $g(x) = x^6 - 5x^3 + 2$, $j(x) = f(x)g(x)$

$$j'(x) = f'(x)g(x) + f(x)g'(x)$$

$$= (2x)(x^6 - 5x^3 + 2) + (x^2 + 1)(6x^5 - 15x^2)$$

The Quotient Rule: $j(x) = \dfrac{f(x)}{g(x)}$

$j'(x) = \lim \quad \frac{f(x+h)}{g(x+h)} - \frac{f(x)}{g(x)} \qquad \lim \quad f(x+h)g(x) - f(x)g(x+h)$

$$j'(x) = \lim_{h\to 0} \frac{\frac{f(x+h)}{g(x+h)} - \frac{f(x)}{g(x)}}{h} = \lim_{h\to 0} \frac{\frac{f(x+h)g(x) - f(x)g(x+h)}{g(x)g(x+h)}}{h}$$

$$= \lim_{h\to 0} \frac{f(x+h)g(x) - f(x)g(x) + f(x)g(x) - f(x)g(x+h)}{h\cdot g(x)g(x+h)}$$

$$= \lim_{h\to 0} \frac{1}{g(x)g(x+h)} \left(\lim_{h\to 0} g(x) \cdot \frac{f(x+h)-f(x)}{h} - \lim_{h\to 0} f(x) \cdot \frac{g(x+h)-g(x)}{h} \right)$$

$$= \frac{1}{g(x)^2} \cdot \left(g(x) \cdot f'(x) - f(x) \cdot g'(x) \right)$$

$$= \frac{f'(x)g(x) - f(x)g'(x)}{g(x)^2}$$

e.g. $f(x) = x^2+1$, $g(x) = x^3$, $j(x) = \frac{f(x)}{g(x)}$

$$j'(x) = \frac{(2x)(x^3) - (x^2+1)(3x^2)}{(x^3)^2} = \frac{2x^4 - 3x^4 - 3x^2}{x^6} = \frac{-x^4 - 3x^2}{x^6}$$

$$= \boxed{\frac{-x^2-3}{x^6}}$$

Practice:

Find:

1) $\frac{d}{dx}(x^3 \cdot x^2) = (3x^2)(x^2) + (x^3)(2x^1) = 3x^4 + 2x^4 = \boxed{5x^4}$

Find:

1) $\frac{d}{dx}\left(x^3 \cdot x^2\right) = (3x^2)(x^2)+(x^3)(2x^1) = 3x^4+2x^4 = \boxed{5x^4}$

2) $\frac{d}{dx}\left(\frac{3x+1}{x}\right) = \frac{(3)(x)-(3x+1)(1)}{x^2} = \frac{3x-3x-1}{x^2} = \boxed{\frac{-1}{x^2}}$

3) $\frac{d}{dx}\left(x^{-5}\right) = \frac{(0)(x^5)-(1)(5x^4)}{x^{10}} = \frac{-5x^4}{x^{10}} = \boxed{-5x^{-6}}$

The Power Rule (Again)

$$\frac{d}{dx}(x^{-n}) = \frac{d}{dx}\left(\frac{1}{x^n}\right) = \frac{(0)(x^n)-(1)(nx^{n-1})}{(x^n)^2} = \frac{-nx^{n-1}}{x^{2n}} = \frac{-n}{x^{n+1}}$$

$$= -nx^{-n-1}$$

So $\frac{d}{dx}(x^n) = nx^{n-1}$ is true even if n is negative!

Practice:

Find:

1) $\frac{d}{dx}\left(\frac{x(x+1)}{x+2}\right)$

Let $f(x) = x(x+1)$ $\quad f'(x) = (1)(x+1)+(x)(1) = 2x+1$

$g(x) = x+2$ $\quad g'(x) = 1$

So $j'(x) = \frac{(2x+1)(x+2) - x(x+1)(1)}{(x+2)^2} = \frac{2x^2+5x+2-x^2-x}{(x+2)^2}$

$= \frac{x^2+4x+2}{}$

$$= \boxed{\frac{x^2+4x+2}{(x+2)^2}}$$