

Review for the Final Exam

11/29/2005

In the Land of Oz example, change the transition matrix by making R an absorbing state. This gives

$$\mathbf{P} = \begin{array}{c} \text{R} \\ \text{N} \\ \text{S} \end{array} \begin{array}{ccc} \text{R} & \text{N} & \text{S} \\ \left(\begin{array}{ccc} 1 & 0 & 0 \\ 1/2 & 0 & 1/2 \\ 1/4 & 1/4 & 1/2 \end{array} \right) . \end{array}$$

Find the fundamental matrix \mathbf{N} , and also \mathbf{Nc} and \mathbf{NR} . Interpret the results.

Assume that, at that time, 80 percent of the sons of Harvard men went to Harvard and the rest went to Yale, 40 percent of the sons of Yale men went to Yale, and the rest split evenly between Harvard and Dartmouth; and of the sons of Dartmouth men, 70 percent went to Dartmouth, 20 percent to Harvard, and 10 percent to Yale. Then the Markov chain has the transition matrix

$$P = \begin{array}{c} \text{H} \\ \text{Y} \\ \text{D} \end{array} \begin{array}{ccc} \text{H} & \text{Y} & \text{D} \\ \left(\begin{array}{ccc} 1 & 0 & 0 \\ .3 & .4 & .3 \\ .2 & .1 & .7 \end{array} \right) .$$

Find the probability that the grandson of a man from Dartmouth went to Harvard.

Consider the Markov chain with general 2×2 transition matrix

$$P = \begin{pmatrix} 0 & 1 \\ 1/3 & 2/3 \end{pmatrix}.$$

- Is P ergodic?
- Is P regular?
- If it is ergodic, find the fixed probability vector.

Ty Manegold has observed that in the video game *Nibbles*, the moments of time at which dots appear are Poisson-distributed, with red dots averaging 5 per minute and yellow dots 2 per minute.

1. What is the probability that no yellow dot appears in the next 2 minutes?
2. What's the probability that exactly 3 red dots occur in the next minute?
3. What's the probability that the next dot will appear will be yellow?

One of the dice in a box of 6 was misprinted, having no “3” and instead, two sides with “2”. A die is taken randomly from this box and tossed twice.

1. What is the probability of getting at least one “3”?
2. Given that you did *not* get any 3's, what is the probability that you drew the defective die?

Let c be a constant and X and Y two random variables with finite range. For each of the following statements, say *exactly* when the statement would be true.

1. $E(X + Y) = E(X) + E(Y)$.

2. $V(X + Y) = V(X) + V(Y)$.

3. $E(cX) = cE(X)$.

4. $V(cX) = cV(X)$.

Suppose you roll a die 200 times. Every time a get 2, 3, 4 or 6 you win one dollar, and every time you roll 1 or 5 you lose two dollars.

1. What is the probability that you earn more than 10 dollars?
2. What is the probability that you lose more than 10 dollars?
3. If you keep rolling, what is the probability that you will win, on the average, at least a quarter?

Consider flipping two fair coins. Let $X = 1$ if the first coin is heads, and $X = 0$ if the first coin is tails. Let $Y = 1$ if the two coins show the same thing (i.e. both heads or both tails), with $Y = 0$ otherwise. Let $Z = X + Y$, and $W = XY$.

1. What is the probability function of Z ?
2. What is the probability function of W ?
3. What is $E(Z)$ and $E(Y)$?