MATH 20 HW # 1 DUE 10/3/08 SECTION 1.2

(1.) $P(\emptyset) = 0$ $P(\{a,b\}) = \frac{1}{2} + \frac{1}{3} = \frac{5}{6}$

$P(\{a\}) = \frac{1}{2}$ $P(\{b,c\}) = \frac{1}{3} + \frac{1}{6} = \frac{1}{2}$

$P(\{b\}) = \frac{1}{3}$ $P(\{a,c\}) = \frac{1}{2} + \frac{1}{6} = \frac{2}{3}$

$P(\{c\}) = \frac{1}{6}$ $P(\Omega) = 1$

(4.) (a.) HEADS ON FIRST TOSS

(b.) ALL HEADS OR ALL TAILS

(c.) EXACTLY ONE TAIL

(d.) NOT ALL HEADS

(5.) (a.) $\frac{1}{2}$ (c.) $\frac{3}{8}$ (WE ASSUME UNIFORM DISTRIBUTION IN #5, i.e. A FAIR COIN.)

(b.) $\frac{1}{4}$ (d.) $\frac{7}{8}$

(6.) $m(n) = an$ AND $\sum_{n=1}^{6} m(n) = 1$,

THUS $\sum_{n=1}^{6} an = 1$, SO $a \sum_{n=1}^{6} n = 21a = 1$,

SO $a = \frac{1}{21}$. THUS $m(n) = \frac{1}{21} n$,

SO THAT $P(\{2,4,6\}) = \frac{2}{21} + \frac{4}{21} + \frac{6}{21} = \boxed{\frac{4}{7}}$.

(7.) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= 1 - P(\tilde{A}) + P(B) - P(A \cap B) = 1 - \frac{1}{3} + \frac{1}{2} - \frac{1}{4} = \boxed{\frac{11}{12}}$.

(8.) $\Omega = \{a, g, p\}$. $m(a) = m(p) = \frac{1}{2} m(g)$

AND $m(a) + m(p) + m(g) = 1$. SOLVING, $m(a) = m(p) = \frac{1}{4}$

AND $m(g) = \frac{1}{2}$, THUS $P(\{a\}) = P(\{p\}) = \frac{1}{4}$ AND $P(\{g\}) = \frac{1}{2}$.

(10.) $\Omega = \{\text{NEITHER, HOUSE, SENATE, BOTH}\}$

$P(\{\text{HOUSE, BOTH}\}) = \frac{6}{10}$

$P(\{\text{SENATE, BOTH}\}) = \frac{8}{10}$

$P(\{\text{HOUSE, SENATE, BOTH}\}) = \frac{9}{10}$

SO $P(\{\text{BOTH}\}) = \frac{6}{10} + \frac{8}{10} - \frac{9}{10} = \boxed{\frac{1}{2}}$ BY THEOREM 1.4.

(11.) (a.) $P(\text{ACE}) = \frac{4}{52} = \frac{1}{13}$. $\boxed{1:12}$.

(b.) $P(\{HH\}) = \frac{1}{4}$. $\boxed{1:3}$.

(c.) $P(\{(6,6)\}) = \frac{1}{36}$. $\boxed{1:35}$

(WE ASSUME UNIFORM DISTRIBUTION IN #11, i.e. FAIR DECK, COIN AND DICE.)

(12.) 3 : 1 ODDS IS A PROBABILITY OF $\boxed{\frac{3}{4}}$.

(13.) $P(\{\text{ROMANCE}\}) = \frac{2}{5}$

$P(\{\text{DOWNHILL}\}) = \frac{1}{3}$

THUS $P(\{\text{ROMANCE, DOWNHILL}\}) = \frac{2}{5} + \frac{1}{3} = \frac{11}{15}$,

SO THE ODDS THAT ROMANCE OR DOWNHILL WINS IS $\boxed{11:4}$.

(15.) $\Omega = \{AA, AB, AC, BA, BB, BC, CA, CB, CC\}$

$P(\{BA, BB, BC\}) = \frac{3}{10}$, $P(\{AB, BB, CB\}) = \frac{4}{10}$

AND $P(\{BB, BC, CB\}) = \frac{1}{10}$, THUS

$P(\{AB, BA, BB\}) = \frac{3}{10} + \frac{4}{10} - \frac{1}{10} = \boxed{\frac{3}{5}}$ BY THE LAWS.

(19.) $P(A \cup B \cup C) = P((A \cup B) \cup C)$

$= P(A \cup B) + P(C) - P((A \cup B) \cap C)$

$= P(A) + P(B) - P(A \cap B) + P(C) - P((A \cap C) \cup (B \cap C))$

$= P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C)$

$+ P(A \cap B \cap C)$, BY REPEATED APPLICATION

OF THEOREM 1.4.

(21.) $P(\{n\}) = \frac{1}{2^n}$, SO

$P(\{10\}) = \frac{1}{2^{10}} = \frac{1}{1{,}024}$

$P(\{11\}) = \frac{1}{2^{11}} = \frac{1}{2{,}048}$

$P(\{12\}) = \frac{1}{2^{12}} = \frac{1}{4{,}096}$.

$P(\{10, 11, 12\})$
$= P(\{10\}) + P(\{11\}) + P(\{12\})$
$= \boxed{\frac{7}{4{,}096}}$.

(23.) WE MUST SHOW $m \geq 0$ AND $\sum_{j=1}^{\infty} m(j) = 1$.

i.e. $m(j) \geq 0$ FOR ALL $j = 0, 1, 2, \ldots$,

CLEARLY $m \geq 0$,

SINCE $0 < r < 1$,

NOW, $\sum_{j=1}^{\infty} m(j) = \sum_{j=0}^{\infty} (1-r)^j r = \frac{r}{1-(1-r)} = 1$

(GEOMETRIC SERIES).

(26.) $P(\text{HIGHER}) + P(\text{LOWER}) + P(\text{SAME}) = 1$ SINCE $\Omega = \text{HIGHER} \cup \text{LOWER} \cup \text{SAME}$ (DISJOINT PAIRWISE)

ALSO, $P(\text{HIGHER}) = P(\text{LOWER})$ BY SYMMETRY, THUS

$P(\text{HIGHER}) = \frac{1 - P(\text{SAME})}{2} = \frac{1 - \frac{3}{51}}{2} = \boxed{\frac{8}{17}}$.

SECTION 3.1

(1.) $4! = 24$

(2.) $4 \cdot 3 = 12$

(3.) 2^{32} (ABOUT 4.29 BILLION)

(5.) $3^2 = 9$

$3 \cdot 2 = 6$

(7.) $\frac{4}{5} \cdot \frac{3}{5} \cdot \frac{2}{5} \cdot \frac{1}{5} = \frac{24}{625}$

(15.) $\Omega = \{(a_1, \ldots, a_n) : a_i = 1, 2 \text{ OR } 3\}$.

LET E BE THE EVENT THAT EXACTLY ONE PROCESSOR RECEIVES NO JOBS. THEN $E = E_1 \cup E_2 \cup E_3$ (PAIRWISE DISJOINT) WHERE E_K IS THE EVENT THAT PROCESSOR K RECEIVES NO JOB, AND BOTH OTHERS DO. THUS $E_K = \{(a_1, \ldots, a_n) \in \Omega : a_i \neq K\}$
$- \{(l, \ldots, l), (m, \ldots, m)\}$,

WHERE l, m ARE THE NUMBERS 1, 2, 3 NOT EQUAL TO k.

THUS $|E_K| = 2^n - 2$,

SO $P(E) = P(E_1 \cup E_2 \cup E_3) = P(E_1) + P(E_2) + P(E_3)$

$= 3\left(\frac{2^n - 2}{3^n}\right) = \boxed{\frac{2^n - 2}{3^{n-1}}}$.

(17.) $P(n) = 1$ IF $n \geq 13$, AND 0 IF $n = 1$.

FOR $n = 2, \ldots, 12$, $P(n) = 1 -$ PROBABILITY OF NO COMMON B-DAY MONTH.

$$= 1 - \left(\frac{11}{12} \cdot \frac{10}{12} \cdot \ldots \cdot \frac{13-n}{12} \right) = 1 - \frac{(12)_n}{12^n}$$

$$= \boxed{1 - \frac{12! / (12-n)!}{12^n}}$$