1.(3 points) Prove the following binomial identity: $\binom{2n}{n} = \sum_{j=0}^{n} \binom{n}{j}^2$.

Hint: Consider an urn with n red balls and n blue balls inside. Show that each side of the equation equals the number of ways to choose n balls from the urn.

2.(2 points) Find two sequences of positive real numbers, $\{a_n\}$ and $\{b_n\}$, such that $a_n \sim b_n$ but $a_n^n \nsim b_n^n$.

Bonus (+2 points) Prove: $\binom{2n}{n} < 4^n$. Do NOT use Stirling's formula. Your proof should be just one line.