MATH 20, WORKSHEET 4

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Due Friday October 6th

(1) Let X be a discrete random variable that takes only positive integer values. Our normal formula for the expected value of X says

$$E[X] = \sum_{k=1}^{+\infty} kP(X = k).$$

Prove the following alternate formula:

$$E[X] = \sum_{k=1}^{+\infty} P(X \geq k).$$

(2) Compute the expected value of the geometric distribution with parameter p.

(3) Recall that if $P(A) > 0$, then $X|A$ is a random variable where

$$m_{X|A}(x) = P(X = x|A) = \begin{cases} \dfrac{m_X(x)}{P(A)}, & \text{if } x \in A \\ 0, & \text{otherwise.} \end{cases}$$

Hence,

$$E[X|A] = \sum_{x \in \Omega} x \cdot P(X = x|A).$$

(a) Assume that $P(\text{not } A) > 0$. Write $E[X]$ in terms of $E[X|A]$ and $E[X|\text{not } A]$. (Hint: see the formula below)

(b) Assume that $F_1, F_2, \ldots, F_k$ are events with positive probability, pair wise disjoints and $\Omega = F_1 \cup F_2 \cup \cdots \cup F_k$. Show that

$$E[X] = \sum_{j=1}^{k} E[X|F_j]P(F_j).$$