Probability Practice Final Exam

Exam 6:00–8:00 on Saturday, December 8

1 In the Florida election there were 6 million voters, and the difference in number of votes between the two candidates was about 500. Assume that each voter was equally likely to choose Bush or Gore, and determine the likelihood that their final totals would be within 500 of each other.

This is a job for the central limit theorem. The mean is 3 million and the standard deviation is roughly 1225. If the difference between them is 500 then the measured proportion of Gore voters is 250 away from the mean, which is roughly 0.2 standard deviations. The likelihood of this is about 15.86%.

2 Assume that the *actual* voter preference was that 500 more people prefer Bush to Gore. Using the technique of Section 9.1, how many people would you have to poll in order to predict the outcome of the election with 95% certainty? Explain your result.

The difference between their proportional vote is $2 \cdot \frac{1}{24,000}$, so we need to find n large enough that

$$P\left(\left|A_n - \frac{1}{2}\right| > \frac{1}{24,000}\right) < 0.05.$$

In other words, we need n large enough that $\frac{1}{24,000}$ is about 2 standard deviations. The standard deviation of A_n is $\frac{1}{2\sqrt{n}}$ so we need $n \geq 576,000,000$. This shows that the assumptions used in the example (namely, the number of people is small relative to the total population) do not hold in this election.

3 In a Markov chain the next state is determined only by the current state. Suppose we have an experiment that is more complicated. There are four coins

- Coin 1 has probability $\frac{1}{5}$ of H
- Coin 2 has probability $\frac{2}{5}$ of H
- Coin 3 has probabitity $\frac{3}{5}$ of H
- Coin 4 has probability $\frac{4}{5}$ of H. At each step, we choose a coin to flip based on the current flip and the previous flip, according to the rule:
- HH, choose Coin 1
- HT, choose Coin 2
- TH, choose Coin 3
- TT, choose Coin 4. Make this experiment into a Markov chain in which the states are the possible lists of what happened last time and what happened this time: $S = \{HH, HT, TH, TT\}$. Write down the transition matrix. Is this Markov chain absorbing?

Using the states in the order given, the transition matrix is

$$\begin{pmatrix} \frac{1}{5} & \frac{1}{5} & 0 & 0 \\ 0 & 0 & \frac{2}{5} & \frac{3}{5} \\ \frac{3}{5} & \frac{2}{5} & 0 & 0 \\ 0 & 0 & \frac{4}{5} & \frac{1}{5} \end{pmatrix}$$

This chain is not absorbing, since there are no absorbing states.

4 Generalize number 3. Suppose I have a procedure in which the next outcome depends on the current outcome and each of the k outcomes that came before. Show how to think of this as a Markov chain. Describe the states of your Markov chain explicitly, and explain what the transition matrix would look like.

Suppose the original list of states is $S = \{s_1, \ldots, s_r\}$. The new list of states is

$$T = \{(x_1, x_2, \ldots, x_{k+1}) \mid x_i \in S\}.$$

We need to be given a rule which tells us for any given list of states

$$(x_1, x_2, \ldots, x_{k+1})$$

what the probability that the next state will be s_i. Let's call this probability

$$p_i((x_1, x_2, \ldots, x_{k+1})).$$

If the current state (in T) is $(x_1, x_2, \ldots, x_{k+1})$ then the next state will be

$$(x_2, x_3, \ldots, x_{k+1}, s_i)$$

with probability $p_i((x_1, x_2, \ldots, x_{k+1}))$. The resulting matrix will have a *lot* of zeros in it.

5 Problems 14, 19 in Section 6.1

6 Problems 8, 15 in Section 6.2

7 Problem 9 in Section 8.1

8 Problems 4, 12 in Section 9.1

9 Problems 7, 8, 13, 14 in Section 9.2

10 Problem 9, 12 in Section 11.2