Name: ____________________________

Math 20 Midterm 1

October 19, 2011

Instructions: This is a closed book, closed notes, computer-free exam. You are not to give nor to receive help from any outside source during the exam. Remember that your instructors can clarify any questions that are not clear to you.

Please show all of your work and justify all of your answers.

Honor Statement:

I have neither given nor received help on this exam, and all of the answers are my own work.

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Signature

Question	Points	Score
1	18	
2	10	
3	12	
4	10	
5	10	
6	10	
7	5	
8	3	
Total:	78	

1. Short Answer:

Answer each of the following questions. You do not need to show any work. (3 points each)

(a) [3 points] Let A, and B be events such that $P(A \cup B) = 11/12$, $P(\tilde{A}) = 1/3$ and $P(B) = 1/2$. What is $P(A \cap B)$?

(b) [3 points] Let A and B be independent events with $P(A) = .4$ and $P(B) = .5$. What is $P(A \cup B)$?

(c) [3 points] The Dartmouth Bridge Club is trying to pick officers: a president, vice-president and secretary. Assuming there are 12 members including Zach and Sam. How many ways can this be done if Zach and Sam can't both be officers?

(d) [3 points] If you roll 5 (fair, 6-sided) dice, what is the probability you get two distinct pairs and one other value (a two pair)?

(e) [3 points] Having broken out of your cell and stolen the warden's set of keys, you are one door away from getting out of prison. Assume there are n keys total, each equally likely to be the correct key. How many do you expect to have to try before finding the right key (you can keep track of which keys you have already used)?

(f) [3 points] In your frantic rush to the outside world, you fail to keep track of which keys you try on the last door. In this case, how many keys do you expect to try (there are still n keys)?

2. [10 points] Officials at Pineapple Air have noticed that about 4% of ticketed passengers do not show up. The new guy on staff has an unheard of idea: over-booking the flights! Given their planes seat 108 passengers and 111 tickets are sold, what is the probability that some one gets left off the Pineapple Express flight?

3. Let X be the outcome from rolling a (fair) 4-sided die and Y be a random integer between 1 and X (so for $X = 3$, we have Y drawn uniformly at random from $\{1, 2, 3\}$).

   (a) [4 points] Find the joint distribution for (X, Y)

   (b) [4 points] Find the marginal distribution for Y

   (c) [4 points] Find the conditional distribution for X given $Y = k$ for $k = 1, 2, 3, 4$.

4. [10 points] For a certain disease, Dr. Jill will recommend surgery if she thinks her patient has an 80% or greater chance of being ill. Currently, she thinks her patient Dan has a 60% chance of being ill. She is about to receive the results from a test that is almost always positive for those with the disease (you can assume there are no false negatives). Unfortunately, there is a 30% chance that someone without the disease but with diabetes will test positive (a false positive), and Dan is diabetic. If the test is positive, will Dr. Jill recommend surgery?

5. We each have \$2. Flip a (fair) coin. When it comes up heads, you give me a \$1. For tails, I give you \$1. We flip until one of us has all the money.

   (a) [5 points] What is the expected length of the game?

   (b) [5 points] Given the first flip is heads, what is the chance you win?

6. For random variables X and Y with $E(X) = \mu$ and $E(Y) = \nu$, we define the covariance of X and Y to be

$$\mathrm{Cov}(X, Y) = E((X - \mu)(Y - \nu))$$

(a) [4 points] What is $\mathrm{Cov}(X, X)$?

(b) [6 points] For X and Y independent, show $\mathrm{Cov}(X, Y) = 0$.

7. [5 points] There are two bent coins on the table in front of you. Let p_1 be the probability that the first coin flips heads and p_2 be the probability that the second flips heads. These values are unknown (and presumably different). If you want to flip two heads in a row, are you better off picking a coin at random and flipping twice or flipping each coin once?

8. [3 points] BONUS: An amoeba will divide into two identical amoebas $3/4$ of the time and die the remaining $1/4$. What is the probability that the amoeba's lineage will never die off?