Math 20, Midterm 2
October 16th

Name _____ (please print)

Instructions

- Please **print your name** in the blank space above.
- Please **turn off cell phones** or other electronic devices which may be disruptive.
- Calculators or other computing devices are not allowed.
- Except when indicated, you must show all work and give justification for your answer. **A correct answer with incorrect work will be considered wrong.**

All work on this exam should be completed in accordance with the Dartmouth Academic Honor Principle.

TIPS:

- Work cleanly and neatly; this makes it easier to give partial credit.
- Use scratch paper to figure out your answers and proofs before writing them on your exam.
- Please box your answers, when appropriate.
- You dont have numerically expand all answers. For example, you can leave an answer in the form $5! \cdot \binom{7}{2} \cdot \binom{10}{3}$, rather than 302400.
- Consider signing the FERPA waiver:

FERPA waiver: By my signature I relinquish my FERPA rights in the following context: This exam paper may be returned en masse with others in the class and I acknowledge that I understand my score may be visible to others. If I choose not to relinquish my FERPA rights, I understand that I will have to present my student ID at my instructors office to retrieve my examination paper. FERPA waiver signature: _____

Grader's use only:

1. _____ /14

2. _____ /15

3. _____ /21

4. _____ /30

Total: _____ /80

Section 1: True or False

1. (14 points) Choose **True** or **False**. *No justification is required for your answers. No partial credit will be awarded.*

   (a) $m_X(j) = \dfrac{1}{10}$ is a distribution function for $j = 0, 1, \ldots, 10$.

   True False

   (b) If X_1 and X_2 are independent random variables and c is a constant, then
   $$E[X_1 + cX_2] = E[X_1] + cE[X_2].$$
   True False

   (c) If X_1 and X_2 are independent random variables, then $V[X_1 - X_2] = V[X_1] + V[X_2]$.

   True False

   (d) A fair coin is tossed repeatedly till it comes up heads. The probability that heads occurs for the first time on the third toss, given that it didn't occur on the first toss is 1/4.

   True False

   (e) Twenty couples go the movies. Assume that their seats were randomly assigned to two rows of twenty seats each. The probability that no couple sits in the same row is
   $$\binom{40}{20} 2^{-20}.$$
   True False

   (f) $E(X^2) \geq E(X)^2$.

   True False

   (g) The number of indistinguishable ways to split a group of 25 people in five teams is
   $$\frac{25!}{5!5!5!5!5!}.$$
   True False

Section 2: Fill in the blank

2. (15 points) *No justification is required for your answers. There will be little or no partial credit.*

   (a) A couple decides to have children until they either have three children of the same gender, or until they have four children total, whichever comes first. Find the expected number of children they will have.

Answer:

(b) Three cards are chosen at random from a standard deck of 52 cards. What is the probability that 1 of the 3 is the ace of spades?

Answer:

(c) Flip a fair coin 3 times and win j dollars if the longest streak of consecutive heads or tails has length j. What is the expected value and the variance of this game?

Answer:

Section 3: Free response

You must show all work to receive credit!

3. (21 pts) A bus arrives at a bus stop on average every 12 minutes. Assume that bus arrivals can be modeled by a Poisson process.

   (a) What is the probability of 0 buses arriving over a period of 24 minutes?

   (b) What is the probability of at least 3 buses arriving over a period of 48 minutes?

(c) What is the probability of the first bus arriving before t minutes?

(d) 12 minutes have passed, and no bus has arrived. Find the expected value of how much more time you have to wait for the next bus.

4. (30 pts) Pat is required to sell candy bars to raise money for the 6th grade field trip. Pat is not supposed to return home until five candy bars have been sold. So the child goes door to door, selling candy bars. At each house, there is a p probability of selling one candy bar and a $q = (1 - p)$ probability of selling nothing.

   (a) What's the probability of Pat selling his first candy bar at the nth house?

   (b) What's the expected value and variance for the number of houses that Pat needs to visit to sell his first candy bar?

(c) After visiting exactly k houses, what is the probability of Pat selling exactly n candy bars?

(d) What's the expected value and the variance for the number of candy bars sold if Pat visits k houses?

(e) What's the probability of Pat selling his fifth candy bar at the nth house?

(f) What's the expected value and variance for the number of houses that Pat needs to visit to sell 5 candy bars?

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