Central Limit Theorem

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A More General Central Limit Theorem

Theorem. *Let $X_1, X_2, \ldots, X_n, \ldots$ be a sequence of independent discrete random variables, and let $S_n = X_1 + X_2 + \cdots + X_n$. For each n, denote the mean and variance of X_n by μ_n and σ_n^2, respectively. Define the mean and variance of S_n to be m_n and s_n^2, respectively, and assume that $s_n \to \infty$. If there exists a constant A, such that $|X_n| \leq A$ for all n, then for $a < b$,*

$$\lim_{n\to\infty} P\left(a < \frac{S_n - m_n}{s_n} < b\right) = \frac{1}{\sqrt{2\pi}} \int_a^b e^{-x^2/2}\, dx \ .$$

Midterm Review

Let S_n be the number of successes in n Bernoulli trials with probability .8 for success on each trial. Let $A_n = S_n/n$ be the average number of successes. In each case give the value for the limit, and give a reason for your answer.

1. $\lim_{n\to\infty} P(A_n = .8)$.

2. $\lim_{n\to\infty} P(.7n < S_n < .9n)$.

3. $\lim_{n\to\infty} P(S_n < .8n + .8\sqrt{n})$.

4. $\lim_{n\to\infty} P(.79 < A_n < .81)$.

In the middle of the night cars arrive at the I-91 security blockade, just south of White River Junction, at an average rate of 18 cars per hour.

1. What is the probability that no cars arrive between 2:00 am and 2:05 am tomorrow morning?

2. What is the probability that exactly two cars arrive between 2:00 am and 2:05 am tomorrow morning?

3. Over how long an interval would the probability of having no arrivals be $1/e^2$(about 13.5%)?

A bank accepts rolls of pennies and gives 50 cents credit to a customer without counting the contents. Assume that a roll contains 49 pennies 30 percent of the time, 50 pennies 60 percent of the time, and 51 pennies 10 percent of the time.

1. Find the expected value and the variance for the amount that the bank loses on a typical roll.

2. Estimate the probability that the bank will lose more than 25 cents in 100 rolls.

3. Estimate the probability that the bank will lose exactly 25 cents in 100 rolls.

4. Estimate the probability that the bank will lose any money in 100 rolls.

A tourist in Las Vegas was attracted by a certain gambling game in which the customer stakes 1 dollar on each play; a win then pays the customer 2 dollars plus the return of her stake, although a loss costs her only her stake. Las Vegas insiders, and alert students of probability theory, know that the probability of winning at this game is $1/4$. When driven from the tables by hunger, the tourist had played this game 240 times. Assuming that no near miracles happened, about how much poorer was the tourist upon leaving the casino? What is the probability that she lost no money?

A manufactured lot of buggy whips has 20 items, of which 5 are defective. A random sample of 5 items is chosen to be inspected. Find the probability that the sample contains exactly one defective item

1. if the sampling is done with replacement.

2. if the sampling is done without replacement.

Suppose that you have a Bernoulli trial with probability of success $p = 0.3$. Let $X_j = 1$ if the jth outcome is a success and 0 if it is a failure, S_n the sum of the first n of them.

1. What does the Law of Large Numbers tell you about S_n/n?

2. Why can you apply the Law of Large Number for Bernoulli trials?

A survey is conducted to determine the proportion of people who will purchase a new video game. Estimate the sample size needed to be at least 99% confident that the estimated proportion will be within two percentage points of the true value?

A die is thrown until the first time the total sum of the face values of the die is 700 or greater. Estimate the probability that, for this to happen,

1. more than 210 tosses are required.

2. less than 190 tosses are required.

A fair coin is tossed 10,000 times independently. If you win a dollar if H occurs and lose a dollar if T occurs, what is the probability that you win at least \$200?