

Math 20: Discrete Probability

Exam 1

October 25, 2000

Rules Each page counts equally toward your Exam grade. All answers should be explained: a numerical answer won't be worth full credit.

- 1 Let X be a random variable with sample space $\Omega = \{a, b, c, d, e, f\}$.
- (a) Write down a distribution function for X which gives each outcome a different probability.
- (b) With your distribution, are the events $A = \{a, b, c\}$ and $D = \{d, e, f\}$ independent?
- (c) With your distribution, are the events $E = \{a, b, c, d\}$ and $F = \{c, d, e, f\}$ independent?
- (d) If $E \subseteq F$ can it happen that E and F are independent? Explain.

2(a) Find the probability of a bad Hearts hand. A bad hearts hand has 13 cards, with the following conditions

2 of the top 3 spades

no other spades

3 of the top 5 hearts

2 of the bottom 5 hearts

no other hearts

all other cards are from the top 5 in their suit.

2(b) Find the probability of a frustrating poker hand. A frustrating poker hand involves 8 cards, as follows. In the first 5 cards, there is

two adjacent cards in one suit (suit X)

a card in some other suit that matches one of the previous two

cards in the remaining two suits which do not match any of the previous cards.

The next 3 cards, together with the paired card in suit X, make a four card straight in suit X. (For simplicity, you may use wrap-around straights such as K, A, 2, 3.)

3 Bob has a pretty good poker face, at least when he doesn't get a bad hand. If his hand is good, he doesn't show any reaction 90% of the time. If his hand is middling, he never shows any reaction. But if his hand is bad, he shows a negative reaction half the time. Let's say the probability of a good hand is $\frac{1}{4}$, the probability of a bad hand is $\frac{1}{4}$, and all the other hands are middling.

(a) If you are playing poker with Bob and he doesn't have any reaction to his hand, what is the probability that he has a good hand?

(b) Suppose you somehow know that he does not have a middling hand. Now what is the probability that he has a good hand?

4 One in ten CDs has manufacturing defects. Quality control finds 99% of the defective CDs before they are packaged and shipped. (I just made these numbers up.)

(a) What is the exact probability that there are at least 3 defective CDs in a shipment of 500 CDs?

(b) What is the Poisson approximation for this probability?

5 Big asteroids hit the Earth every six million years or so, on average (I just made this number up).

(a) What is the probability that two big asteroids will hit the Earth in the next century?

(b) What is the probability that no big asteroids will hit the Earth at any time during the next century?