Midterm Exam
Math 20
October 22nd, 2014

Name: ____

Please read the following instructions before starting the exam:

- This exam is closed book. You may not give or receive any help during the exam, though you may ask the instructors for clarification if necessary.
- Be sure to show all work whenever possible. Even if your final answer is incorrect, an appropriate amount of partial credit can be assigned.
- Please circle or otherwise indicate your final answer, if possible.
- The test has a total of 7 questions, worth a total of 90 points. Point values are indicated for each question.
- You will have two hours from the start of the exam to complete it.
- Good luck!

HONOR STATEMENT: I have neither given nor received help on this exam, and I attest that all the answers are my own work.

SIGNATURE:________________________

This page is for grading purposes only

Problem	Points
1	
2	
3	
4	
5	
6	
7	
Total	

1. **Multiple choice.** Circle the correct answer to each question. [5 points each, no partial credit].

Let X, Y each be picked from [0,1] with uniform density. Which of the following events is independent of $x < y$?

(a) $x < 1/2$

(b) $y < 1/2$

(c) $y < 1 - x$

(d) $y < x$

Let X be a normal random variable with $\mu = 50$ and $\sigma = 10$. Let Z be a standard normal random variable (with $\mu = 0,\ \sigma = 1$.) You have a cumulative distribution table for Z. What is the value of Z that has the same cumulative probability as $X = 65$?

(a) 65

(b) 1

(c) 1.5

(d) 15

You are about to play chess against your friend, and you agree to play 3 games. Suppose that in any game, the probability that you win is 0.6, the probability that your friend wins is 0.4 and the probability of a draw is 0. What is the chance that your friend still wins at least twice?

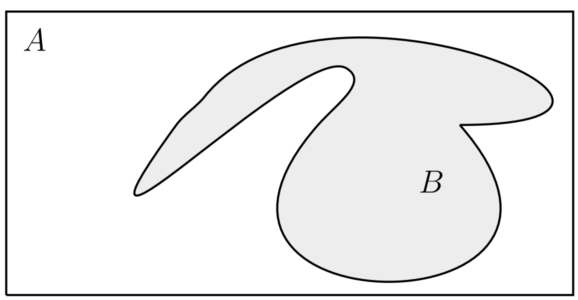
(a) $0.6^2 0.2^4 + 0.2^3$

(b) $\binom{3}{2} 0.6^2 0.4$

(c) $\binom{3}{2} 0.6^2 0.4 + 0.6^3$

(d) $\binom{3}{2} 0.4^2 0.6 + 0.4^3$

Rectangle A, pictured, has area equal to some quantity a and region B has area equal to some quantity b. If you sample 100 points in rectangle A uniformly, how many of the sampled points, on average, do you expect to be in region B?



(a) $100/a$

(b) $100 \times b$

(c) $100 \times b \times a$

(d) $100 \times b/a$

2. A coin comes up "tails" with probability p on any particular flip. Let the random variable X be the number of flips until, and including, the first time it comes up "tails."

   - What is the probability distribution of X? [5 points]
   - What is $E(X)$? [5 points]

3. Recall the *Dirac delta*. One of the many ways to describe it is as the following limit of functions:

$$\delta(x) = \lim_{a\to 0} \delta_a(x)$$

$$\delta_a(x) = \frac{1}{a\sqrt{\pi}} \mathrm{e}^{-x^2/a^2}.$$

To do this problem, you won't need the functions $\delta_a(x)$ as long as you remember the properties of their limit. Otherwise, you may use them to guide you.

Give a formula and draw a graph for the cumulative distribution function $F(x)$ of $f(x) = \delta(x - 5)$. [10 points]

4. There are 21 balls in the urn, 6 of them are blue, 7 are red, and 8 are yellow. If you pick 5 balls from the urn at random, what is the probability that x of them will be blue, and y of them will be red, for any $x,\ y$? [10 points]

5. What is the probability, that if you pick a permutation of $\{1, 2, \ldots, 10\}$ uniformly at random, at least one of 1, 2, and 3 will be a fixed point? [10 points]

6. Make 3 marks on a unit stick, picking the location of each of them uniformly at random. What is the probability density of the position of the mark that is furthest to the left (i.e. closest to 0)? [15 points]

7. Let X, Y and Z each be a random variable describing a result of rolling a different fair six-sided die. Calculate:

   - $E(X + Y + Z)$
   - $E(X \times Y \times Z)$
   - $V(X + Y + Z)$

   [15 points]