

NAME: _____

SECTION: _____

MATH 20 MIDTERM 1

July 20, 2009

INSTRUCTIONS: This is a closed book, closed notes exam. You are not to provide or receive help from any outside source during the exam.

- You may use a calculator, but you may not use any Stat programs included in your calculator
- Show all of your work.

HONOR STATEMENT:

I have neither given nor received help on this exam, and all of the answers are my own.

Signature

Question	Points	Score
1	8	
2	12	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
Total:	100	

1. [8 points] Let A and B be events such that $P(A \cap B) = \frac{1}{4}$, $P(\bar{A}) = \frac{1}{3}$ and $P(B) = \frac{1}{2}$. What is $P(A \cup B)$? Note: \bar{A} means that complement of A .

2.

- (a) [5 points] Many students think that in a true-false exam, each question has a $\frac{1}{2}$ chance of being true and a $\frac{1}{2}$ of being false. Following this model, what is the probability that in a 10 question true-false exam, exactly 5 of them are true. Also calculate the probability of all ten questions being true.

- (b) [7 points] Professor K likes to be sneaky about the way he selects his questions. The ten question exam could be thought of as a word consisting of letters T and F, there are 2^{10} such words. He decides that every word with one T will be equally likely, while a word with 2 T's will be twice as likely and with 3 T's, three times as likely, etcetera. In other words, the probability is proportional to the number of truths in the "word". What is the probability that 5 of the ten questions will be true? What about that all ten questions are true?

3. [10 points] Mr. K wants to know what dance to teach, so he makes a survey of his 25 students. He finds out that 12 students know how to dance salsa, 15 students know how to dance merengue and 5 students know bachata. The five bachata dancers know at least one of the other two dances. Only two students that know salsa don't know merengue. Only one student that knows bachata doesn't merengue and 3 students dance all three. How many students don't dance either of these three Caribbean dances?

4. [10 points] What is the probability that if you throw six dice, at least one number will appear exactly once? Example: 1, 2, 2, 2, 3, 4 satisfies the conditions because 1 appears exactly once (note 3 and 4 also appear exactly once) while 2, 2, 2, 3, 3, 3 doesn't because the two numbers that appear appear more than once.

5. [10 points] Alice and Bob play a game. Alice rolls a regular die (six sides) and Bob rolls a a four-sided die (that is a die with values 1, 2, 3, 4). The winner of the game is whoever rolled higher, in case of a tie, Bob wins. Who is most likely to win? What is the probability of Alice winning?

6. [10 points] PokerStars, the biggest online poker casino, has a weekly \$215 tournament which attracts thousands of players. A poker player with the screenname MLG managed the feat of winning two times in 2005, something that seems very unlikely. Assuming that every week, the same 1000 players play the tournament and that each one of them has the same probability of winning the tournament, compute the probability of a player winning two weeks in a year to show that this “feat” is not unlikely.

7. Rafael Nadal has an impressive record on clay courts. From 2005 to 2009 he is 123-6 for a 95.34% winning percentage. Roger Federer is also impressive in clay courts with a record of 84-14 for a 85.71%. His record against non-Nadal players is 82 – 5 for a 94.25% winning percentage. Nadal’s record against non-Federer players is 114 – 4 for a 96.61% winning percentage. These numbers are a little complicated, so we will simplify them to try to find the probability of each player winning the French Open:

- (a) [5 points] The French Open is in clay courts. To reach the final a player must win six matches. Assume that both Nadal’s and Federer’s probabilities of winning a match are 0.95 and that they won’t play each other before the final. What is the probability that they will meet in the final?

- (b) [5 points] Nadal has beaten Federer 9 times while losing 2 times in clay, giving him a 81.81% winning percentage. To make it simpler, assume Nadal beats Federer with probability 0.8. Assuming Federer and Nadal won't play each other unless they play at the final, compute the probability that Federer will win the French Open. (Note that he had to win six matches before reaching the final and the final might or might not have Nadal in it).

8. [10 points] A doctor gives a test to a patient for a particular disease, which we shall call D . D is a rare disease that appears in 1 out of every 500 people. Experience has shown that 97% of the cases in which the patient has D the test is positive, and in 98% of the cases in which the patient does not have D , the test comes out negative. If the test comes out positive, what is the probability that the patient has disease D ?

9. [10 points] Leunig's Bistro, a restaurant in downtown Burlington, had the following deal in March 2009: At the end of the meal you would roll five dice and if you would get five a kind, the meal was free. If you would not roll five of a kind, you would receive as percent discount the sum of what you rolled. Example, if you rolled 3, 2, 4, 2, 1, the sum is 12, hence you get 12% discount. What is the expected percent discount? (Note: If you roll five of a kind the discount is 100%, because the meal is free, don't forget to take this into account when computing the expectation.)

10. [10 points] Alice is playing poker against Bob. There are \$100 in the pot and Bob bets the rest of his money, which is \$200. Alice can either CALL to win \$300 if she wins the hand and lose \$200 if she loses the hand, or FOLD not winning or losing anything. Bob accidentally shows his hand and Alice with her knowledge of probability figures out that she has 44.75% of winning the hand. Compute the expected value to determine whether she should CALL or FOLD.

BONUS (5 points): Name at least three important figures in the history of probability for five points.