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You may discuss the problems and solutions with anyone but the work written up and submitted must be done on your own. Justify every step. Give credit where it is due.

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1. Simplify the following expressions.  $n, d$  are integers. Write your final expression in the most succinct form possible.

(a)  $\frac{(d+n)!}{n!}$

(b)  $\frac{(2n+2)!}{(2n)!}$

(c) Write  $\frac{((3!)!)!}{2!}$  as  $k \times n!$ . Find  $k$  and  $n$

2. Prove directly:  $a, b$  are positive integers. If  $a$  and  $b$  are consecutive integers then  $a + b$  is odd.
3. Prove the above statement using the contrapositive. First write the contrapositive statement.
4. What is wrong with the following proof? We prove that  $1 = 2$ .

$$\begin{aligned} a = b &\implies a^2 = ab \\ &\implies a^2 + a^2 = a^2 + ab \\ &\implies 2a^2 = a^2 + ab \\ &\implies 2a^2 - 2ab = a^2 + ab - 2ab \\ &\implies 2a^2 - 2ab = a^2 - ab \\ &\implies 2(a^2 - ab) = a^2 - ab \\ &\implies 2 = 1 \end{aligned}$$

You may use the product notation  $\prod_{i=1}^k a_i$  which means  $a_1 \times a_2 \cdots \times a_k$  or the sum notation  $\sum_{i=1}^k a_i$  which means  $a_1 + a_2 + \cdots + a_k$  where needed.