Problem Session 1 – July 3, 2012

You may work in groups of 2 or 3, or alone. Please make sure everyone's name is written on the page you turn in. Please write neatly and show all work. There is scrap paper available if needed. This assignment will be graded credit/no-credit.

Problem: Suppose we have an urn with one red ball and one blue ball. Each time we draw a ball from the urn, we replace the ball and add a ball of the same color to the urn. (So for example, on our first turn, if we draw a red ball, then we would return that ball and add a red ball to the urn so that our urn would now contain 2 red balls and one blue ball.)

After n draws, what is the probability that we have added k red balls to the urn?

For $k > n$, $P(k \text{ red balls}) = 0$

For $0 \le k \le n$,

note $P(\underbrace{RRR\ldots R}_{k}\underbrace{BB\cdots B}_{n-k}) = \dfrac{(1\cdot 2\cdot 3\cdots k)(1\cdot 2\cdots\cdots(n-k))}{2\cdot 3\cdot 4\cdots(k+1)(k+2)\cdots\cdots(n+1)} = \dfrac{k!(n-k)!}{(n+1)!}$

the prob. that we draw k red balls 1st, then $n-k$ blue balls

denominator is increasing b/c we add one ball each time we draw

Notice that by changing the order of the draws, the denominator remains the same (still adding 1 ball each time we draw) & in the numerator we multiply the same #s in a different order. So for every possible way of drawing k red balls & $n-k$ blue balls, the probability is $\dfrac{k!(n-k)!}{(n+1)!}$. Since there are $\binom{n}{k}$ ways to choose k red balls & $n-k$ blue balls (choose k of the n draws to be red) then the probability of drawing k red balls (= prob that we added k red balls) is $\binom{n}{k}\dfrac{k!(n-k)!}{(n+1)!} = \dfrac{n!}{(n-k)!k!}\cdot\dfrac{k!(n-k)!}{(n+1)!} = \dfrac{1}{n+1}$. So the answer does <u>not</u> depend on k!