Problem Session 6 – August 21, 2012

You may work in groups of 2 or 3, or alone. Please make sure everyone's name is written on the page you turn in. Please write neatly and show all work. There is scrap paper available if needed. This assignment will be graded credit/no-credit. You only need to turn in ONE of these problems (all parts).

Problem 1: April is preparing to take a qualifying exam (not multiple choice) in Probability. There is a list of 75 questions posted on the department's website. The exam consists of 5 of these questions, of which she must answer four correctly in order to pass.

(a) April goes into the exam knowing how to answer 65 of the questions. Should she expect to pass? (Find how many questions she should expect to be able to answer. This value should not contain a sum.)

(b) Find the exact probability that April passes (you do not need to simplify your answer).

(c) Given that the exam contains one problem she doesn't know how to do, what is the probability that she will pass?

(d) Given that she passes the exam, what is the probability that the exam contained one problem she didn't know how to do?

(e) April and Bob studied for this exam together. If you assume that they will definitely score within 1 point of each other, what is the probability that Bob will pass the exam, given that April passed?

Problem 2: Consider this variation on the Ehrenfest model with four balls, where now, if the left urn is either completely empty or completely full, it stays that way. So the states where the left urn is empty or full are absorbing states.

(a) What is the transition matrix P for this chain?

(b) Confirm that $N = \begin{bmatrix} 2.5 & 3 & 1.5 \\ 2 & 4 & 2 \\ 1.5 & 3 & 2.5 \end{bmatrix}$ is the fundamental matrix of P.

(c) If we start with 3 balls in the left urn, what is the probability that the chain will eventually be absorbed in the state where the left urn is empty?

(d) If we start with 3 balls in the left urn, how many steps do we expect the chain to take until being absorbed, that is, until the left urn is either completely empty or completely full?

Problem 3: Suppose you have two dice, a red and a white one. The red one is loaded so that the probability of getting 6 is 1/3 and the remaining outcomes are equally likely among themselves. The white die is fair.

(a) You take the red die and your opponent takes the red die. The dice are rolled and the player with the highest result wins. In the case of equal results, the white die wins. Which die does one choose to have a better chance of winning.

(b) What is the expected value and variance for the outcome of the red die?

(c) Roll the red die. What is the probability that in 100 rolls, you roll exactly 30 6's?

(d) Roll the red and white dice together. What is the probability that in 100 rolls you roll 12 exactly 10 times?

Problem 4: A gin hand consists of 10 cards from a deck of 52 cards.

(a) What is the probability that all 10 cards are of the same suit?

(b) What is the probability that the hand has a 4,3,2,1 distribution of suits?

(c) Let $X =$ the number of hearts in your gin hand. What is $E(X)$ and $V(X)$?

(d) Given that the hand has five hearts and five spades, what is the probability that it has exactly 2 aces?

(e) Given that the hand has at least one ace, what is the probability that it has exactly two aces?

Problem 5: You flip n coins.

(a) If the coins are fair what is the probability that the proportion of heads is between 0.45 and 0.55 if $n = 100$? What about if $n = 10000$?

(b) If the coins are not fair, but $p = 1/5$ is the probability of flipping heads, what is the probability that the proportion of heads is between 0.2 and 0.3 if $n = 100$?

(c) Suppose that you play a game with a fair coin where if X is the number of heads that show when you flip it n times, then you win $Y = 2^X$ dollars. How much should you have to pay to play this game in order for the game to be fair?

(d) Calculate the variance for Y.