Homework 7 – Due August 15, 2012

Be sure to write your first and last name on your homework. Please write neatly and staple all pages together. You should show all your work! Note there are TWO pages to this homework.

1. [Section 11.1, Problem 2] In the "telephone" example (see your notes or example 11.4 in the book), let $a = 0$ and $b = 1/2$. Find P, P^2, P^3. What is P^n? What happens to P^n as $n \to \infty$? Interpret this result.

2. For the drunkard's walk problem (see notes or example 11.13 in your book), assume now that the probability that the man takes a step to the right (towards the bar) is 2/3 and the probability that he takes a step to the left is 1/3.

   (a) What is our transition matrix P? Write this transition matrix in canonical form.

   (b) Find N, $t = Nc$, and $B = NR$.

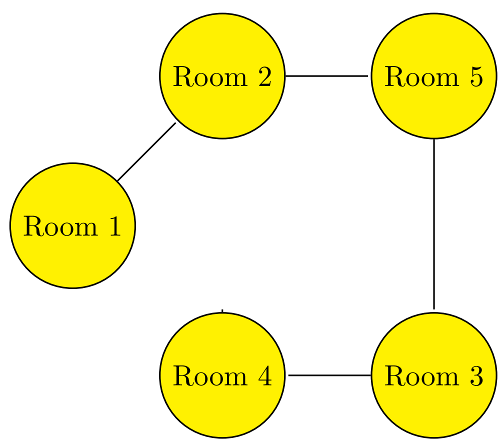
   (c) If the drunkard is dropped off on corner 2, what is the expected number of times he'll visit corner 3?

   (d) If he is dropped off on corner 1, what is the expected number of steps until he ends up at the bar or at home?

   (e) If he is dropped off on corner 1, what is the probability he'll end up at the bar?

   (f) Suppose he is dropped off at either corner 1, 2, or 3 with equal probability. What is the probability he'll end up at the bar?

3. Suppose there is a drunk taking a walk through a museum. This museum has five rooms. At each step he leaves the room he is in and enters an adjacent room. Each possibility is equally likely. The graph below represents the museum. If he is in room 5, there is a 1/2 chance he'll enter room 2 on the next step and a 1/2 chance he'll enter room 3 on the next step.



   (a) Write the transition matrix for this scenario.

   (b) Is this Markov chain absorbing? Is it ergodic? Is it regular?

(c) Do problem number 83 of the online homework. Upload your code. Write down the probability vector you get that describes the probability that the drunk is in each of the five rooms after 30 steps. What about after 31 steps?

4. Explain the proof of theorem 11.6 in words (the proof in symbols is in your book). Your answer must include an explanation of the first line of the proof in your book: "$\mathbf{B}_{ij} = \sum_n \sum_k q_{ik}^{(n)} r_{kj}$".

5. [Section 11.2, Problem 18] Assume that a student going to a certain four-year medical school in northern New England has, each year, a probability q of flunking out, a probability r of having to repeat the year, and a probability p of moving on to the next year (in the fourth year, moving on means graduating).

   (a) Form a transition matrix for this process, taking as states $F, 1, 2, 3, 4$, and G, where F stands for flunking out and G stands for graduating, and the other states represent the year of study. Write your transition matrix in canonical form.

   (b) For the case, $q = .1, r = .2$, and $p = .7$, find the time a beginning student can expect to be in their second year. how long should this student expect to be in medical school?

   (c) Find the probability that this beginning student will graduate.