This will be an evolving document of good probability problems. Skip around throughout the document, sometimes similar problems are all grouped together. Some problems here may be harder than can be reasonably asked during a midterm, but are good practice nevertheless (use your best judgement). If you see any errors, confusing sentences or have anything to add, please let me know!

- Basic set theory (union, intersection, complement, subset, disjoint, $(A \cup B)^C = A^C \cap B^C)$).
- Events, random variables, distribution functions, outcome space, probability.
- Counting with "options", eg. binary words, permutations, ...
- Basic probability equalities; Law of equally likely outcomes.
  eg. if A and B are disjoint, $P(A \cup B) = P(A) + P(B)$.
- Binomial coefficients and Pascal's Relation - $\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}$; Binomial Theorem.
- Choosing colored balls from boxes; birthday problem.
  eg. What is the probability draw you draw one yellow ball and three red balls?
- Number of ways to stand in a line (permutations) and counting with order.
- Binomial distribution and Bernoulli trials process.
- Coin flipping and rolling dice problems.
- Conditional probability; independent events/random variables.
- Application of conditional probability to diagnostic testing.
- Paradoxes and Monty Hall (can you explain Monty Hall to a friend?).
- Expectation, variance and standard deviation; linearity of expectation, and the set of theorems that tell us how independence can allow us to break things apart more.
  i.e. If X and Y are independent $E(XY) = E(X)E(Y)$.
- Different types of distributions: bionmial, geometric, negative binomial, hypergeometric, Poisson. Know how to recognize them, when to use each one and how calculate their expecatation, (and for binomial, know the variance).
- Working with indicator random variables and decomposing random variables into smaller pieces; coupon collecting.
- Sums of random variables.
- Markov's Inequality and Chebyshev's Inequality.
- And anything else you can think of.

The first 14 questions listed here are from the textbook *A First Course in Probability* by Sheldon Ross (in chapters 2- 4).

1. Suppose that A and B are disjoint events for which $P(A) = .3$ and $P(B) = .5$. What is the probability that a) either A or B occurs; b) A occurrs but B does not; c) both A and B occur?

2. If 8 rooks are randomly placed on a chessboard, compute the probability that none of the rooks can capture any of the others. That is, compute the probability that no row or column contains more than one rook.

3. Two symmetric dice have both had two of their sides painted red, two painted black, one painted yellow, and the other painted white. When this pair of dice are follwed, what is the probability they both land on the same color?

4. Two cards are chosen at random from a deck of 52 cards. What is the probability that a) they are both aces; b) have the same value?

5. There are n socks, 3 of which are red, in a drawer. What is the value of n if when 2 of the socks are chosen randomly, the probability that they are both red is $\frac{1}{2}$.

6. There are ten yellow balls and four red balls in an urn. You take three balls from the urn. What is the probability that you got one red ball and two yellow balls?

7. An urn contains 5 white balls and 10 black balls. A fair die is rolled and that number of balls is randomly chosen from the urn. What is the probability that all of the balls selected are white? What is the conditional probability that the die landed on 3 if all the balls selected are white?

8. There are 15 tennis balls in a box, of which 9 have not previously been used. Three of the balls are randomly chosen, played with, and then returned to the box. Later, another 3 balls are randomly chosen from the box. Find the probability that none of these balls has ever been used.

9. In a class there are 4 freshman boys, 6 freshman girls, and 6 sophomore boys. How many sophomore girls must be present if sex and class are to be independent when a student is selected at random?

10. A true-false question is to be posed to a team on a quiz show. Both people on the team will, independently, give the correct answer with probability p. Which of the following is a better strategy for the team?
    (a) Choose one of them and let that person answer the question; or
    (b) have them both consider the question and then either give the common answer if they agree or, if they disagree, flip a coin to determine which answer to give?
11. In the previous question, if $p = .6$, and the couple uses the second strategy, what is the conditional probability that the couple gives the correct answer given that a) they agree, or b) disagree?
12. Two athletic teams play a series of games; the first team to win 4 games is declared the overall winner. Suppose that one of the teams is stronger than the other and wins each game with probability .6, independent of the outcomes of the other games. Find the probability that the stronger team wins the series in exactly i games. Do this for $i = 4, 5, 6, 7$. How does this compare the probability of the better team winning a best-2-out-of-3 game series?
13. Two balls are chosen at random from an urn containing 8 white, 4 black, and 2 orange balls. Suppose that we win 2 points for each black ball selected and we lose 1 point for each white ball selected. Let X denote our winnings. What are the possible values for X, and what are the probabilities associated with each value? What is our expected earnings?
14. A newsboy purchases papers at 10 cents and sells them at 15 cents. However, he is not allowed to return unsold papers. If his daily demand is a binomial random variable with $n = 10$, $p = \frac{1}{3}$, approximately how many papers should he purchase as to maximize his expected profit?
15. A restaurant has 100 tables, each night they take 120 reservations with the knowledge that each individual table has an 80 percent chance of showing up. What is the probability that the restaurant has taken too many reservations?
16. I have a jar with 27 pennies and 3 nickels. I reach into the jar and take out a coin. If it is a penny, I return it to the jar and keep drawing. What is the expected number of draws I must make until I draw a nickel?
17. There is class of 10 students; Alex and Barney are best friends in the class. The students stand in line. What is the probability that Alex is not first or

Barney is not last in line? What is the probability that Alex is in line ahead of Barney?

18. In a class of 100 people, 25 of them have a younger sibling. We ask 20 students if they have a younger sibling, what is the probability that exactly 10 of them have a younger sibling?

19. There are 10 people on an elevator and 15 floors in a building. What is the probability that everyone gets off at a different floor?

20. Can you determine the expectation and variance of all the distributions we have seen in class?

21. We roll a die twice. Let A be the event that the first die was a 4. Let B be the event that the sum of the dice is 5. Let C be the event that the sum of the dice is 7. Let D be the event that the second die was a 3. For each pair of events, decide whether the two events are independent, disjoint or neither.

Sketched-up Answers:

2. Compare to a re-arrangement. 4. $\frac{\binom{4}{2}}{\binom{52}{2}}, \frac{13\binom{4}{2}}{\binom{52}{2}}$. 6. $\frac{\binom{4}{1}\binom{10}{2}}{\binom{14}{3}}$. 7. Look through conditional probability notes. 8. Conditional probability, some balls could have been played with twice. 9. Nine sophomore girls. 10. Both strategies produce the same. 11. Conditional probability. 12. Use binomial $p = .6$ to write down few probabilities, then calculate expectation by hand. 13. Case-by-case. 14. Case-by-case, $n = 3$ is the one that works out (it is not enough to say n is the expected demand, must consider his profit, which we really need to check a few values nearby, eg. if the amount lost is low, he might buy-up extra newspapers on the off-chance that more than 3 wanted papers). 15. Binomial, $P(X > 100)$. 16. Geometric distribution. 17. $1-P(\text{Alex is first})-P(\text{Barney is last})+P(\text{Alex first and Barney last})$. 1/2 18. Hypergeometric distribution, choosing from a small population balls-in-bucket type. 19. Compare to birthday problem. 21. A and B are neither, A and C are independent, B and C are disjoint. A and D are independent (different dice).