You may work together on these problems for the rest of today. Please write up a solution to Problem 1 or Problem 2 for the next weekly problem set. You may submit a correct and nicely presented solution of Problem 4 or Problem 5 for one bonus point each on your quiz. Please write the names of classmates you worked on these problems with on the top of your page. *You may not consult outside sources of any kind (aside from one another) when solving these questions and submitting your solutions for extra credit.*

1. There are three prisoners and three hats. Each prisoner is assigned a random hat, either blue or red. Each person can see the hats of two others, but not their own. On a cue, they each must guess their own hat color or pass. All three must say their answer simultaneously, so there is no way to base their guesses on the guesses of others. The three prisoners win release if at least one person guessed correctly and none guessed incorrectly (passing is neither correct nor incorrect). What is the best strategy? What are the probability that they win using this strategy?

2. In rural Russia in the old days, children used to play the following game to divine an answer to a yes or no question. Someone would take three pieces of ribbon, which together have 6 ends, and would hold the ribbons so that the person can only see the ends. Then the asker would tie three pairs of ends together. If the result is a single loop, the answer is yes. What is the probability of a single loop?

3. You are a prisoner sentenced to death. The Emperor offers you a chance to live by playing a simple game. He gives you 50 red balls, 50 blue balls and 2 empty bowls. He then says: "Divide these 100 balls into these two bowls. You can divide them in any way you like as long as you use all the balls. Then I will blindfold you and mix the bowls around. You then choose one bowl and remove one ball. If the ball is blue you will live, but if the ball is red, you will die." How do you divide the balls up so that you have the greatest probability of drawing a blue ball?

4. A hundred people get on a plane with a hundred seats. Unfortunately, the first passenger through boarding is a mathematician trying to solve a very absorbing problem, and she manages to lose her boarding pass on the way from the gate to the plane. Once on the plane, she has no idea which is her seat. She picks a seat uniformly at random. Now, every passenger that comes in will pick their seat if it's empty, and if it's not, will pick an empty seat uniformly at random.

What is the probability that the last person on board sits in their own seat? *Hint*: Try playing out this game several times with, say, 6 people on the plane.

5. The princess Sleeping Beauty participates in an experiment that starts on Sunday. She is told that she will be put to sleep, and while she is asleep a fair coin will be tossed that will determine how the experiment will proceed. If the coin comes up heads, she will be awakened on Monday, interviewed, and put back to sleep, but she won't remember this awakening. If the coin comes up tails, she will be awakened and interviewed on Monday and Tuesday, again without remembering either awakening. In either case, the experiment ends when she is awakened on Wednesday.

   Whenever Sleeping Beauty is awakened and interviewed, she won't know which day it is or whether she has been awakened before. During each awakening, she is asked: "What is your degree of certainty/belief that the coin landed heads?" What should she respond?