

1. A company makes spools of thin wire to sell to manufacturers. Occasionally, a small kink will form in the wire and compromise the strength of the wire; this spool of wire must be discarded and re-formed. Suppose approximately 2 percent of the spools of wire made by this process must be discarded. In a box of 350 spools, we would expect about $np = 350(0.02) = 7$ to have defects. Use the Poisson approximation to calculate the probability that between 5 and 8 spools must be discarded in a pallet of 350.
2. Assume that the probability that there is a significant accident in a car factory during one year's time is .001. If a country has 100 factories, estimate the probability that there is at least one such accident during a given year. Use the Poisson approximation.
3. Suppose that X is a Poisson random variable with parameter λ . Show that $V(X) = \lambda$. *Hint*: First find $E(X)$ and $E(X(X-1))$. Even though we talked about it in class, please show the steps for deriving $E(X)$.
4. Suppose that X is a Poisson random variable with parameter λ . Show that $P(X = k)$ is maximized when $k = \lfloor \lambda \rfloor$, where $\lfloor \lambda \rfloor$ is the greatest integer less than or equal to λ (eg. $\lfloor 3.14 \rfloor = 3$).
Note: This means that the most probable outcome is the value, $k = \lfloor \lambda \rfloor$.
5. A fair coin is tossed 100 times. Calculate the expectation and standard deviation, σ , for the number of heads. What does Chebyshev's Inequality tell us about the probability that the number of heads that turn up deviates (i.e. is further away) from the expected number of heads by three or more standard deviations?
6. Let X be a random variable with $E(X) = 0$ and $V(X) = 1$. What is a (reasonable) integer value k that will assure us that $P(|X| \geq k) \leq .01$?
7. In a class of 60 students, the professor calls on one student chosen at random per class. There are 30 class periods in a term.
 - (a) Write a formula for the exact probability that a given student is called upon k times during the term.
 - (b) Write a formula for the Poisson approximation for this probability. Using your formula, estimate the probability that a given student is called upon more than twice (i.e. called on three or more times).

8. There are n students in a class and there are n chairs in the room. Anticipating the beginning of the school year, Miss Apple wrote down a seating assignment. Instead, on the first day of class, the students arrive happy and excited and sit down wherever they like - she never had a chance to ask them to sit in their assigned seats at all. What is the expected number of students sitting in the seat she had given them on her seating assignment?