1. Suppose you choose a real number X from the interval $[2, 10]$ with a density function of the form $f(x) = Cx$, where C is a constant.
   (a) Find C.
   (b) Find $P(E)$, where $E = [a, b]$ is a subinterval of $[2, 10]$.
   (c) Find $P(X > 5)$ and $P(X^2 - 12X + 35 > 0)$.
   (d) Let $A = [3, 7]$ and $B = [2, 4]$. Find $P(A|B)$.

2. Let X be a random variable with density function
$$f_X(x) = \begin{cases} cx(1-x) & 0 \le x \le 1 \\ 0 & \text{otherwise} \end{cases}$$
   (a) What is the value of c?
   (b) What is the cumulative distribution function $F_X(x)$ for X?
   (c) What is the probability that $X < 1/4$?

3. Let X be a continuous random variable given by the density function
$$f_X(x) = \begin{cases} \lambda e^{-\lambda x} & \text{if } x \ge 0 \\ 0 & \text{if } x < 0 \end{cases}$$
   (a) Verify that the $f_X(x)$ is a density function.
   (b) Find the cumulative distribution function $F_X(x)$ for X.

4. Two shoppers split up at a mall and agree to meet back at the main entrance between noon and 1 pm. Each person arrives sometime within this hour and waits for 15 minutes before they leave. What is the probability that they find one another?

5. Choose independently two numbers B and C at random from the interval $[0, 1]$ with uniform density. Note that the point (B, C) is then chosen at random in the unit square.
   (a) Find the probability that $|B - C| < 1/2$.
   (b) Find the probability that both $B < 1/2$ and $1 - C < 1/2$.
   (c) Find the probability that $|B - C| < 1/2$, $B < 1/2$ and $1 - C < 1/2$ (that is, these three events all occur).

6. Take a stick of unit length and break it into three pieces, choosing the break points simultaneously at random. What is the probability that the three pieces can be used to form a triangle? *Hint*: The previous problem should help.

7. You are in a room with square tiles with side lengths $l = 2.5$ inches. You drop a quarter on the ground and notice that the quarter lies completely on one tile (without touching the extremely thin line between the tiles). A standard quarter has diameter $d = .955$ inches. What is the probability that a randomly droped quarter will lie completely on one tile?

8. With the same set up as the previous problem, now suppose that your tiled floor is made up of equillateral triangles with side lengths $l = 4.5$ inches. What is the probability that a randomly dropped quarter will lie completely on one tile? *Note*: I will give nearly-full credit for a good picture and explanation (lacking a complete answer).

9. Let X be a continuous random variable with values uniformly distributed over the interval $[0, 20]$.

   (a) Find the mean and variance of X.

   (b) Calculate $P(|X - 10| \geq 2)$, $P(|X - 10| \geq 5)$, $P(|X - 10| \geq 9)$, and $P(|X - 10| \geq 20)$ exactly. Calculate the bounds given by Chebyshev's Inequality for the same probabilities. How do the bounds compare to the exact values, i.e. are the bounds very good at some times and not as good in others?

Challenge Problem: (3 points extra credit and a good puzzle)

Some number of circles whose circumferences sum to 10 are placed in the unit square. Prove that there exists a line passing through at least 4 of them.

After Wednesday, you can "purchase a hint" for this problem, and a complete solution will be worth 2 points. You may turn in a solution to this problem any time in the next couple of weeks (say, before the next quiz). This is a difficult problem!