1. Choose a number U from the unit interval with the uniform distribution. Find the cumulative distribution function and density function for the random variables:

   (a) $Y = U + 2$,

   (b) $Y = 3U$. (updated)

2. Suppose that the number of years a car will run is exponentially distributed with parameter $\lambda = \frac{1}{4}$. If Prosser buys a used car today, what is the probability that it will still run after 4 years?

3. Suppose that the height, in inches, of a 25-year old man is a normal random varaible with parameters $\mu = 71$ and $\sigma^2 = 6.25$. What percentage of 25-year old men are over 6 feet 2 inches tall? What percentage of men over 6 feet tall are over 6 foot 5 inches?

4. You arrive at a bus stop at 10 o'clock, knowing that the bus will arrive at some time uniformly distributed between 10 and 10:30.

   (a) What is the probability that you will have to wait longer than 10 minutes?

   (b) If at 10:15 the bus has not yet arrived, what is the probability that you will have to wait at least an additional 10 minutes?

5. A man named Juan puts in two new lightbulbs: a 60 watt bulb and a 100 watt bulb. It is claimed that the lifetime of the 60 watt bulb has exponential density with average lifetime 200 hours, $\lambda = 1/200$. The 100 watt bulb also has exponential density but with average lifetime 100 hours, $\lambda = 1/100$. He wonders what the probability is that the 100 watt bulb will outlast the 60 watt bulb.

   If X and Y are two independent random variables with exponential densities $f(x) = \lambda e^{-\lambda x}$ and $g(x) = \mu e^{-\mu x}$, then the probability that X is less than Y is given by
   $$P(X < Y) = \int_0^\infty f(x)(1 - G(x))dx,$$
   where $G(x)$ is the cumulative distribution function for $g(x)$.

   (a) Draw a picture or use a few sentences to explain why this expression gives the probability that $P(X < Y)$.

   (b) Use the integral to verify that $P(X < Y) = \frac{\lambda}{\lambda+\mu}$.

(c) Answer Juan's question: What is the probability that the 100 watt bulb will outlast the 60 watt bulb?

(d) If Juan uses the 100 watt bulb and replaces it with the 60 watt bulb when the first bulb runs out, what is the average total lifetime, $X + Y$, of the two bulbs?

6. A fair die is tossed 10,000 times. Find the expected number of rolls that are threes. Find the probability of rolling 200 or fewer threes; use the normal approximation to the binomial distribution.

7. Let X be a uniform random variable on the interval $[0, 2]$ and Y a uniform random variable on the interval $[0, 3]$. Determine the density function for $Z = X + Y$.

8. What do the Chebyshev's Inequality and the Law of Large Numbers have to say about the probability of getting at least 75 heads when flipping a fair coin 100 times? *Hint*: Use the fact that the binomial distribution is symmetric in order to improve your bound.

9. Find a good approximation to the number n so that, when a fair coin is flipped 10,000 times, the probability of observing between 4930 and n heads is 1/2. *Hint*: Use the normal approximation to the binomial distribution.

10. How many times should you toss a *weighted* coin to be at least 90 percent certain that your estimate (your sample average) of $P(\text{Heads})$ is within .01 of the true value?

11. If you have independent and identically distributed random variables $X_1, X_2, \ldots, X_n$, with finite mean and variance, their distribution is approximately normal (so long as n is reasonably large). The previously posted problem has been cancelled.

12. Suppose that $X_1, \ldots, X_{20}$ are independent random variables with density functions $f(x) = 2x$. Let $S = X_1 + \ldots + X_{20}$. Use the Central Limit Theorem to approximate $P(S \leq 10)$.