Math 20 – Lab 1 (due July 23)

Each experiment asks you to perform some simulations and describe the results. When you're asked to explain your findings, I'm not looking for long paragraphs; just a couple of sentences will do. Please type your responses (use LaTeX, if you know how, or else you can use Word or another text editor, though it surely won't look as nice!); your responses should be written in the same document and titled `Math 20 Lab 1 - [your first and last name]`.

Each experiment also asks you to submit code. You must name the files as instructed and submit them with your completed assignment. Please *comment your code* so that I know what you're trying to do.

Please send everything to me (one .docx/.doc/.pdf and two .R files) **by email** by the beginning of class on Monday, July 23.

1. Today is Archer's 35th birthday. His friends have baked him a cake and placed on it 35 lit candles. After the traditional ritual singing, Archer tries to blow out the candles. Each time he blows, he successfully extinguishes between 1 and n of the remaining candles (inclusive), where n is the number of candles that are still lit. Each of these n outcomes is equally likely.

   For example, after his first attempt, there are somewhere between 0 and 34 candles that remain lit, and each of these occurs with equal probability; he had a 1/35 chance of blowing out just one, 1/35 of blowing out two, etc. After his second attempt (if necessary), there are somewhere between 0 and 33 candles that remain lit (though these are no longer equiprobable events).

   (a) Write an R program that simulates this experiment. Run this simulation many times (at least 10,000), recording the number of attempts needed to extinguish all of the candles. Summarize your findings in a table, and include some kind of plot to visualize this data. Describe your findings.

   (b) Edit the previous R program so that you can input the age of the person as a variable n. Pick a few values of n (they don't have to actually be ages of any humans!) and perform the simulations from part (a) again. Again describe your findings and include some sort of plot.

   (c) Use the code from part (b) to do the following: For each n in the set $\{1, 2, \ldots, 100\}$, run the simulation many times (at least 10,000). Plot all the results together in some way. (There are many options on how to do this. See the link on the course website for ideas, and pick one that you think works well for this kind of problem.) What seems to be the behavior of E(number of attempts) as n increases?

   (d) Submit your code for this problem in a file named `candles-yourlastname.R`. Make sure you've commented your code so I understand what it does and how to run it.

2. Cyril, a casino mogul, has invented a new game. The problem is that he has no idea what the expected payout is, so he doesn't know what to charge for it! The game works like this: the player starts with a score of 0. The computer generates a random real number between 0 and 1, inclusive. The number is added to the player's score and the player gets $10. If the player's score is still below 1, this is repeated–the computer picks a random real number between 0 and 1, adds it to the player's score, and the player gets $10. The game ends when the player's score is 1 or greater.

   For example, suppose the computer outputs the numbers 0.2153, 0.561148, 0.392327. In this case, the player wins $30, and a third number is not generated; the game stops because the total score is 1 or larger.

   Note: R's native `runif(n)` will create a list of n randomly selected number between 0 and 1.

   (a) Write a simulation that plays this game. Run it many times (at least 10,000) and take the estimate the expected payoff of the game by averaging your results. What is this number? Describe your findings and include some sort of plot.

   (b) What if, instead, the real number returned by the computer is squared before it's added to the player's score? Repeat part (a) with this change. Now what is the expected payoff? Describe your findings and include some sort of plot.

   (c) Invent some other variant of the game (add a rule, change the payout structure, etc.) and simulate the expected payout now. Describe your findings and include some sort of plot. Be creative! While you're at it, come up with a snappy name for this game.

   (d) Submit your code for this problem in a file named `casino-yourlastname.R`. Make sure you've commented your code so I understand what it does and how to run it.