Math 20 – Problem Set 2 (due July 11)

This problem set is due at the *beginning* of class. This is just the problem list; please work out these problems on a different sheet of paper. Please write neatly, staple the pages together, and explain your work where appropriate. You do not need to simplify binomial coefficients $\binom{n}{k}$ for both which $k > 3$ and $n - k > 3$, or exponentials n^k where $n + k > 8$.

1. Suppose two cards are drawn from a standard 52-card playing deck without replacement. Determine which of the following pairs of events are independent. Justify your answers using the independence equation, not just intuition!

   (a) A : The first card is a spade.
       B : The second card is a club.

   (b) C : The first card is black (a spade or a club).
       D : The second card is a club.

   (c) E : The first card is a face (Jack, Queen, or King).
       F : The second card is a face.

2. In Yahtzee, five fair 6-sided dice are rolled and points are awarded for different combinations. A *large straight* is a result in which the five dice take the distinct values $\{1, 2, 3, 4, 5\}$ or $\{2, 3, 4, 5, 6\}$. What is the probability of rolling a large straight on a single roll of the five dice?

3. In chess, rooks can move any distance either vertically or horizontally, but not diagonally, and if one piece can move to the square of another piece, it can "attack" that piece. Suppose 8 rooks are randomly placed on a (8-by-8) chessboard with at most one rook per square. What is the probability that none of the rooks can attack another? (*Hint: How many ways can the rooks be placed on the board? What must the board look like if no rook can attack another?*)

4. Two archers, Jack and Diane, each shoot a single arrow at a target. Suppose Jack hits the target 60% of the time and Diane hits the target 90% of the time.

   (a) What is the probability that both archers hit the target? What assumption are you making?

   (b) Making the same assumption as above, what is the probability that exactly one of the archers hits the target?

   (c) Under the same assumption, if exactly one of the two arrows hits the target, what is the probability it was Jack who fired this arrow?

   (d) Instead of the assumption you made for parts (a)-(c), suppose instead that Diane fires first, and if she hits the target, Jack's probability of hitting the target drops to 30%, since Jack might hit Diane's arrow. Now what is the probability Jack hits the target after Diane takes her shot?

5. Show that for any two events A and B with $P(A) > 0$ and $P(B) > 0$, if $P(A|B) > P(A)$, then also $P(B|A) > P(B)$. (*Hint: How are these four quantities related?*)

6. (a) Box A contains 4 red balls and 4 blue balls, and Box B contains 2 red balls and 4 blue balls. One of the two boxes is selected at random with equal probability, and then a ball is drawn randomly from the selected box. If the ball is red, what is the probability that the selected box was Box A?

   (b) Suppose now that two balls are drawn from the same box selected in part (a), without replacement. If both balls are red, what is the probability that the selected box was Box A?