Math 20 – Problem Set 3 (due July 18)

This problem set is due at the *beginning* of class. This is just the problem list; please work out these problems on a different sheet of paper. Please write neatly, staple the pages together, and explain your work where appropriate. You do not need to simplify binomial coefficients $\binom{n}{k}$ for both which $k > 3$ and $n - k > 3$, or exponentials n^k where $n + k > 8$.

1. Someone offers you the following game: You roll a fair six-sided die. If you roll a 1, you win $25. If you roll a 2, you win $5. If you roll a 3, you win nothing. If you roll a 4 or a 5, you lose $10. If you roll a 6, you lose $15. Should you play this game? Why or why not?

2. Suppose n bar patrons each check their hats at the door. When they leave, intoxicated, they each take a hat at random. What is the expected number of patrons who leave with the same hat they were wearing when they arrived? How does this number depend on n? (There's an easy way to do this, and there's a hard way...)

3. If 20 indistinguishable balls are allocated randomly into 10 differently-colored boxes in such a way that all allocations are equally likely (eg. having 20 balls in the first box is just as likely as having 20 balls in the third box, and both are just as likely as having 2 in each box), what is the probability that no box is empty? Explain your answer. (This was problem #12 on your exam.)

4. In class on Friday we (hopefully) discussed the triangular distribution that results from rolling 2 fair 6-sided dice and taking their sum. Write an R program to simulate rolling 4 fair 6-sided dice and taking their sum. Do this at least 10000 times and make a table of your results. Then visualize your results using R's `plot()` function. Explain in a sentence or two how this distribution compares to the triangular distribution. Please print your code and output and attach it with this problem set.