MATH 20 – PROBLEM SET 5 (DUE AUGUST 1)

This problem set is due at the *beginning* of class. This is just the problem list; please work out these problems on a different sheet of paper. Please write neatly, staple the pages together, and explain your work where appropriate. You do not need to simplify binomial coefficients $\binom{n}{k}$ for both which $k > 3$ and $n - k > 3$, or exponentials n^k where $n + k > 8$.

1. Let X be a random variable with range $[-1, 1]$ and let $f(x)$ be its density of X. Find μ_X and σ_X^2, if, for $|x| \leq 1$:

   (a) $f(x) = \frac{3}{4}(1 - x^2)$

   (b) $f(x) = \frac{x+1}{2}$

   (c) $f(x) = \frac{3}{8}(x + 1)^2$

   (This is #2 a,c,d from Grinstead and Snell, page 277.)

2. Let X be a random variable defined on the interval $[0, \pi]$ whose density function is $f(x) = \frac{\sin(x)}{2}$. Compute the CDF $F(x)$. What is $P(\frac{\pi}{6} \leq X \leq \frac{\pi}{2})$?

3. On an average 8-hour school day, 960 people walk into Kemeny Hall. Assume, though this is certainly not the case, that this happens randomly at a constant rate over the 8 hours. What is the probability that exactly 8 people walk into Kemeny Hall within a 10-minute interval during the school day? What is the probability that exactly 48 people walk into Kemeny Hall within an hour?

4. The *half-life* of an isotope is the amount of time it takes for the probability of one isotope to decay into another to be 50%. The time it takes for a Carbon-14 isotope to decay into a Nitrogen-14 isotope is given by an exponential distribution with expected value estimated at 8267 years. Find its half-life to the nearest year. Use a calculator to simplify exponentials and logarithms. (Note: Decay of radioactive particles is probably the most appropriate process to model with the exponential distribution.)

5. Suppose the height of an adult male is given by a normal distribution with expected value 70 inches and standard deviation 4 inches.

   (a) Shaquille O'Neal is 83 inches tall. What proportion of adult males are taller than Shaq? Use a standard normal distribution table.

   (b) Darren Sproles is 66 inches tall. What proportion of adult males are shorter than Darren? Do you need a normal distribution table to answer this question?

6. Prove that if X is a continuous random variable with range $[x_1, x_2]$ and finite expected value μ, then,
$$\text{Var}(X) = E(X^2) - \mu^2.$$

(Note: This is true for *all* discrete and continuous RVs with finite mean and variance, and the proof is almost exactly the same!)