Math 20: Probability
Homework 8

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Due Date: August 28, 2020

Please specify whether you complete the homework independently or cooperate with (get help from) the TA, your classmates, friends, online resources, etc.

For every problem, show the process and necessary details.

A pdf file is required for submission.

Problem 1

4 pts

Chapter 7.1 Exercise 5

Consider the following two experiments: the first has outcome X taking on the values 0, 1, and 2 with equal probabilities; the second results in an (independent) outcome Y taking on the value 3 with probability $\frac{1}{4}$ and 4 with probability $\frac{3}{4}$. Find the distribution of

(a) $Y + X$.

(b) $Y - X$.

Problem 2

4 pts

Chapter 7.2 Exercise 5

Suppose that X and Y are independent and $Z = X + Y$. Find f_Z if

(a)

$$f_X(x) = \begin{cases} \lambda e^{-\lambda x}, & \text{if } x > 0 \\ 0. & \text{otherwise} \end{cases}$$

$$f_Y(x) = \begin{cases} \mu e^{-\mu x}, & \text{if } x > 0 \\ 0. & \text{otherwise} \end{cases}$$

Assume that $\lambda \neq \mu$.

(b)

$$f_X(x) = \begin{cases} \lambda e^{-\lambda x}, & \text{if } x > 0 \\ 0. & \text{otherwise} \end{cases}$$

$$f_Y(x) = \begin{cases} 1, & \text{if } 0 < x < 1 \\ 0. & \text{otherwise} \end{cases}$$

Problem 3

4 pts

Chapter 7.2 Exercise 10

Let $X_1, X_2, \cdots, X_n$ be n independent random variables each of which has an exponential density with mean μ. Let M be the minimum value of the X_j. Show that the density for M is exponential with mean $\frac{\mu}{n}$.

Hint: Use cumulative distribution functions.

Problem 4

3 pts

Chapter 8.2 Exercise 4

Let X be a continuous random variable with values exponentially distributed over $[0, +\infty)$ with parameter $\lambda = 0.1$.

(a) Find the mean and variance of X.

(b) Using Chebyshev's Inequality, find an upper bound for the following probabilities: $P(|X - 10| \geq 2)$, $P(|X - 10| \geq 5)$, $P(|X - 10| \geq 9)$, and $P(|X - 10| \geq 20)$.

Note: Are the bounds all useful?

(c) Calculate these probabilities exactly, and compare with the bounds in (b).

Problem 5

3 pts

Chapter 8.2 Exercise 12

A share of common stock in the Pilsdorff beer company has a price Y_n on the nth business day of the year. Finn observes that the price change $X_n = Y_{n+1} - Y_n$ appears to be a random variable with mean $\mu = 0$ and variance $\sigma^2 = \frac{1}{4}$. If $Y_1 = 30$, find a lower bound for the following probabilities, under the assumption that the X_n's are mutually independent.

(a) $P(25 \le Y_2 \le 35)$.

(b) $P(25 \le Y_{11} \le 35)$.

(c) $P(25 \le Y_{101} \le 35)$.

Problem 6

4 pts

Chapter 9.1 Exercise 3

A true-false examination has 48 questions. June has probability $\frac{3}{4}$ of answering a question correctly. April just guesses on each question. A passing score is 30 or more correct answers. Compare the probability that June passes the exam with the probability that April passes it.

Note: You do not need to get a specific number. It is good enough to use the $\mathrm{NA}(a, b)$ notation we have seen in class.

Problem 7

5 pts

Chapter 9.2 Exercise 6

A bank accepts rolls of pennies and gives 50 cents credit to a customer without counting the contents. Assume that a roll contains 49 pennies 30 percent of the time, 50 pennies 60 percent of the time, and 51 pennies 10 percent of the time.

(a) Find the expected value and the variance for the amount that the bank loses on a typical roll.

(b) Estimate the probability that the bank will lose more than 25 cents in 100 rolls.

(c) Estimate the probability that the bank will lose exactly 25 cents in 100 rolls.

(d) Estimate the probability that the bank will lose any money in 100 rolls.

(e) How many rolls does the bank need to collect to have a 99 percent chance of a net loss?

Note: For part (b), (c) and (d), you can simply use the $\mathrm{NA}(a, b)$ notation.

Problem 8

3 pts

Chapter 9.3 Exercise 11

The price of one share of stock in the Pilsdorff Beer Company (see Problem 5) is given by Y_n on the nth day of the year. Finn observes that the differences $X_n = Y_{n+1} - Y_n$ appears to be independent random variable with a common distribution having mean $\mu = 0$ and variance $\sigma^2 = \frac{1}{4}$. If $Y_1 = 100$, estimate the probability that Y_{365} is

(a) ≥ 100.

(b) ≥ 110.

(c) ≥ 120.

Note: For part (a), the answer can be obtained without using a calculator. For part (b) and (c), you can simply use the $\mathrm{NA}(a, b)$ notation.