The Invertible Matrix Theorem

Let A be a square $n \times n$ matrix. **Then** the following statements are equivalent.

- **a.** A is an invertible matrix
- **b.** A is row equivalent to the $n \times n$ identity matrix
- **c.** A has n pivot positions
- **e.** The columns of A form a linearly independent set
- **h.** The columns of A span $\mathbb{R}^n$

- **m.** The columns of A form a basis of $\mathbb{R}^n$
- **n.** $\operatorname{Col} A = \mathbb{R}^n$
- **o.** $\dim \operatorname{Col} A = n$
- **p.** $\operatorname{rank} A = n$
- **q.** $\operatorname{Nul} A = \{\mathbf{0}\}$
- **r.** $\dim \operatorname{Nul} A = 0$
- **s.** The number 0 is **not** an eigenvalue of A
- **t.** $\det A \neq 0$