The Invertible Matrix Theorem

Let A be a square $n \times n$ matrix. Then the following statements are equivalent.

a. A is an invertible matrix

b. A is row equivalent to the $n \times n$ identity matrix

c. A has n pivot positions

d. The equation $A\mathbf{x} = \mathbf{0}$ has only the trivial solution

e. The columns of A form a linearly independent set

f. The linear transformation $\mathbf{x} \mapsto A\mathbf{x}$ is **one-to-one**

g. The equation $A\mathbf{x} = \mathbf{b}$ has at least one solution for each $\mathbf{b} \in \mathbb{R}^n$

h. The columns of A span $\mathbb{R}^n$

i. The linear transformation $\mathbf{x} \mapsto A\mathbf{x}$ maps $\mathbb{R}^n$ **onto** $\mathbb{R}^n$

j. There is an $n \times n$ matrix C such that $CA = I$

k. There is an $n \times n$ matrix D such that $AD = I$

l. A^T is an invertible matrix

m. The columns of A form a basis of $\mathbb{R}^n$

n. $\operatorname{Col} A = \mathbb{R}^n$

o. $\dim \operatorname{Col} A = n$

p. $\operatorname{rank} A = n$

q. $\operatorname{Nul} A = \{\mathbf{0}\}$

r. $\dim \operatorname{Nul} A = 0$

s. The number 0 is **not** an eigenvalue of A

t. $\det A \neq 0$