

THE INVERTIBLE MATRIX THEOREM

Let A be a square $n \times n$ matrix. Then the following statements are equivalent.

- a. A is an invertible matrix
- b. A is row equivalent to the $n \times n$ identity matrix
- c. A has n pivot positions
- d. The equation $A\mathbf{x} = \mathbf{0}$ has only the trivial solution
- e. The columns of A form a linearly independent set
- f. The linear transformation $\mathbf{x} \mapsto A\mathbf{x}$ is **one-to-one**
- g. The equation $A\mathbf{x} = \mathbf{b}$ has at least one solution for each $\mathbf{b} \in \mathbb{R}^n$
- h. The columns of A span \mathbb{R}^n
- i. The linear transformation $\mathbf{x} \mapsto A\mathbf{x}$ maps \mathbb{R}^n **onto** \mathbb{R}^n
- j. There is an $n \times n$ matrix C such that $CA = I$
- k. There is an $n \times n$ matrix D such that $AD = I$
- l. A^T is an invertible matrix