Math 22 Fall 2004

Linear Algebra with Applications

The Inverse of a Matrix

October 11, 2004

Load the packages for doing Linear Algebra

```
> with(Student[LinearAlgebra]):
Warning, the protected name `.` has been redefined and unprotected
```

Define a matrix we want to invert

```
> A := <<0, 3, -1>|<2, -2, 0>|<1, -5, 1>>;
```

$$A := \begin{bmatrix} 0 & 2 & 1 \\ 3 & -2 & -5 \\ -1 & 0 & 1 \end{bmatrix}$$

Augment it with the identical matrix

```
> A_I := <A | IdentityMatrix(3)>;
```

$$A\_I := \begin{bmatrix} 0 & 2 & 1 & 1 & 0 & 0 \\ 3 & -2 & -5 & 0 & 1 & 0 \\ -1 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

Perform row reductions to transform A into
the Reduced Echelon Form (that is, the identity matrix).

```
> A_I := SwapRows(A_I, 1, 3);
```

$$A\_I := \begin{bmatrix} -1 & 0 & 1 & 0 & 0 & 1 \\ 3 & -2 & -5 & 0 & 1 & 0 \\ 0 & 2 & 1 & 1 & 0 & 0 \end{bmatrix}$$

```
> A_I := AddRow(A_I, 2, 1, 3);
```

$$A\_I := \begin{bmatrix} -1 & 0 & 1 & 0 & 0 & 1 \\ 0 & -2 & -2 & 0 & 1 & 3 \\ 0 & 2 & 1 & 1 & 0 & 0 \end{bmatrix}$$

```
> A_I := AddRow(A_I, 3, 2, 1);
```

$$A\_I := \begin{bmatrix} -1 & 0 & 1 & 0 & 0 & 1 \\ 0 & -2 & -2 & 0 & 1 & 3 \\ 0 & 0 & -1 & 1 & 1 & 3 \end{bmatrix}$$

```
> A_I := MultiplyRow(A_I, 3, -1):
  A_I := MultiplyRow(A_I, 2, -1/2):
  A_I := MultiplyRow(A_I, 1, -1);
```

$$A\_I := \begin{bmatrix} 1 & 0 & -1 & 0 & 0 & -1 \\ 0 & 1 & 1 & 0 & \frac{-1}{2} & \frac{-3}{2} \\ 0 & 0 & 1 & -1 & -1 & -3 \end{bmatrix}$$

```
> A_I := AddRow(A_I, 2, 3, -1):
  A_I := AddRow(A_I, 1, 3, 1);
```

$$A\_I := \begin{bmatrix} 1 & 0 & 0 & -1 & -1 & -4 \\ 0 & 1 & 0 & 1 & \frac{1}{2} & \frac{3}{2} \\ 0 & 0 & 1 & -1 & -1 & -3 \end{bmatrix}$$

Now the last 3 columns should form the inverse matrix of A

```
> A_Inverse := LinearAlgebra[DeleteColumn](A_I, [1 .. 3]);
```

$$A\_Inverse := \begin{bmatrix} -1 & -1 & -4 \\ 1 & \frac{1}{2} & \frac{3}{2} \\ -1 & -1 & -3 \end{bmatrix}$$

Let's check our answer

```
> A.A_Inverse, A_Inverse.A;
```

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

```
>
```