Part I

1. (12) Complete the following **definitions**.

(a) An *elementary row operation* on a matrix is one of the following:

(b) The set $\{\mathbf{v}_1, \ldots, \mathbf{v}_p\}$ of vectors in $\mathbb{R}^n$ is *linearly dependent* if

(c) An $n \times n$ matrix A is *invertible* if

2. (4) Let $T : \mathbb{R}^n \to \mathbb{R}^m$ be a linear transformation. When is T one-to-one?

3. (4) Let A be an $n \times n$ matrix. Give 4 conditions on A, each of which is equivalent to A being invertible (do **not** use the definition).

4. (5) For each of the following, fill in the blank using the appropriate letter from the list below so that the given statement is *always* true. You do **not** need to justify your answer.

(A)	are linearly dependent
(B)	are linearly independent
(C)	span $\mathbb{R}^m$
(D)	span $\mathbb{R}^n$
(E)	none of the above makes the statement always true

(a) If A is an $m \times n$ matrix and the equation $A\mathbf{x} = \mathbf{b}$ is consistent for some $\mathbf{b} \in \mathbb{R}^m$, then the columns of A ____________.

(b) If $\mathbf{v}_1, \ldots, \mathbf{v}_n \in \mathbb{R}^m$ such that the vector equation $x_1\mathbf{v}_1 + \cdots + x_n\mathbf{v}_n = \mathbf{0}$ has only the trivial solution, then $\mathbf{v}_1, \ldots, \mathbf{v}_n$ ____________.

(c) If A is an $m \times n$ matrix, then the columns of A ____________ whenever $m < n$.

(d) Let $T : \mathbb{R}^n \to \mathbb{R}^m$ be a linear transformation with standard matrix A. Then T maps $\mathbb{R}^n$ onto $\mathbb{R}^m$ if and only if the columns of A ____________.

(e) Let A be an $m \times n$ matrix and B an $n \times p$ matrix. If $\mathbf{v} \in \mathbb{R}^p$ such that $B\mathbf{v} \neq \mathbf{0}$ and $(AB)\mathbf{v} = \mathbf{0}$, then the columns of A ____________.

5. (10) For each of the following statements, circle either **T** for "True" or **F** for "False." You do **not** need to justify your answers.

T **F** (a) The equation $A\mathbf{x} = \mathbf{0}$ has the trivial solution if and only if there are no free variables.

T **F** (b) Let A be a 4×4 matrix with columns $\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3, \mathbf{a}_4$. If

$$\mathbf{b} = \pi\mathbf{a}_1 - 17\mathbf{a}_3 + \frac{1}{\sqrt{2}}\mathbf{a}_4,$$

then the equation $A\mathbf{x} = \mathbf{b}$ is consistent.

T **F** (c) If none of the vectors in the set $S = \{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ of vectors in $\mathbb{R}^3$ is a multiple of one of the other vectors, then S is a linearly independent set.

T **F** (d) If $\mathbf{w}$ is a linear combination of $\mathbf{u}, \mathbf{v} \in \mathbb{R}^n$, then $\mathbf{u}$ is a linear combination of $\mathbf{v}$ and $\mathbf{w}$.

T **F** (e) If $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ is a linearly dependent set of vectors in $\mathbb{R}^3$, then $\{\mathbf{v}_1, \mathbf{v}_2\}$ is also linearly dependent.

T **F** (f) If T is a mapping such that $T(\mathbf{0}) = \mathbf{0}$, then T is linear.

T **F** (g) If T is a mapping that is one-to-one, then T is onto.

T **F** (h) If $T : \mathbb{R}^{2005} \to \mathbb{R}^{2006}$ is a linear transformation, then T cannot map $\mathbb{R}^{2005}$ onto $\mathbb{R}^{2006}$.

T **F** (i) If $T : \mathbb{R}^{2006} \to \mathbb{R}^{2005}$ is a linear transformation, then T cannot be one-to-one.

T **F** (j) If A and B are invertible $n \times n$ matrices, then AB is invertible and $(AB)^{-1} = A^{-1}B^{-1}$.

Part II

Your solutions to this part of the exam are due at the beginning of class tomorrow (October 13). Fully justify your solutions, and write them *neatly* on $8.5'' \times 11''$ sheets of paper; if you use more than one sheet of paper, staple your pages together in the upper left-hand corner. Use only a (non-colored) pencil or a pen with either blue or black ink.

You may refer only to your text and class notes. Otherwise, you are to work alone and to neither receive nor provide assistance to anyone else.

6. (7) Let

$$A = \begin{bmatrix} 1 & 2 & -3 & 1 & 1 \\ -1 & -1 & 4 & -1 & 6 \\ -2 & -4 & 7 & -1 & 1 \end{bmatrix}.$$

(a) Compute the reduced echelon form of A.

(b) Write the linear system for which A is the augmented matrix, and find the general solution of that system.

7. (8) Let

$$A = \begin{bmatrix} 1 & 0 & 1 \\ -2 & 1 & -1 \\ 2 & -1 & 2 \end{bmatrix}, \qquad \mathbf{e}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \qquad \mathbf{e}_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \qquad \mathbf{e}_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}.$$

(a) Find the inverse of A.

(b) Use part (a) to solve the systems $A\mathbf{x} = \mathbf{e}_i$ for $i = 1, 2, 3$.

(c) Let $\mathbf{a}_1$, $\mathbf{a}_2$, $\mathbf{a}_3$ be the columns of A, and let $T : \mathbb{R}^3 \to \mathbb{R}^2$ be the linear transformation satisfying

$$T(\mathbf{a}_1) = \begin{bmatrix} -1 \\ 1 \end{bmatrix}, \qquad T(\mathbf{a}_2) = \begin{bmatrix} 0 \\ 2 \end{bmatrix}, \qquad T(\mathbf{a}_3) = \begin{bmatrix} 2 \\ 3 \end{bmatrix}.$$

Find the standard matrix for T.

8. (10) Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be the linear transformation given by

$$T(x_1, x_2, x_3) = (2x_3, 3x_1 + x_2, 2x_1 - 2x_3).$$

(a) Find a matrix A such that $T(\mathbf{x}) = A\mathbf{x}$ for all $\mathbf{x} \in \mathbb{R}^3$.

(b) Is T one-to-one? Is T onto?

(c) Is there a vector $\mathbf{u} \neq \mathbf{0}$ such that $T(\mathbf{u}) = \mathbf{u}$?

9. (5) Let C be a 5×7 matrix and D a 7×5 matrix. Can DC be invertible?

10. (5) Suppose a linear transformation $T : \mathbb{R}^n \to \mathbb{R}^n$ has the property that $T(\mathbf{u}) = T(\mathbf{v})$ for some pair of distinct vectors $\mathbf{u}, \mathbf{v} \in \mathbb{R}^n$. Can T map $\mathbb{R}^n$ onto $\mathbb{R}^n$? Why or why not?

NAME : ______________________________

Math 22

2006-10-12

This exam has two parts. The first part you must complete during class today (October 12). Your solutions to the second part are due at the beginning of class tomorrow (October 13). Your solutions to the second part should be fully justified and *neatly* written on $8.5'' \times 11''$ sheets of paper; if you use more than one sheet of paper, staple your pages together in the upper left-hand corner. Use only a (non-colored) pencil or a pen with either blue or black ink.

You may refer to your text and class notes only on the take-home part of this exam. Otherwise, you are to work alone and to neither receive nor provide assistance to anyone else.

Problem	Points	Score
1	12	
2	4	
3	4	
4	5	
5	10	
6	7	
7	8	
8	10	
9	5	
10	5	
Style Points	5	
Total	75	