Introduction to Proofs

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An integer n is *even* if $n = 2k$ for some integer k.

1. Prove that if n is an even integer, then n^2 is also an even integer.

2. Use induction to prove that for any integer $n \geq 1$,

$$1^2 + 2^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}.$$

3. Let r be any real number not equal to 1. Prove that for any integer $n \geq 1$,

$$1 + r + r^2 + \cdots + r^n = \frac{1 - r^{n+1}}{1 - r}.$$

4. In the following statements, a, b, and c are integers.

   (a) There is a unique integer, denoted $-a$, such that $a + (-a) = 0 = -a + a$.

   (b) The integer 1 satisfies $a \cdot 1 = a = 1 \cdot a$.

   (c) The integer 0 satisfies $a \cdot 0 = 0 = 0 \cdot a$.

   (d) The operations $\cdot$ and $+$ satisfy $a \cdot (b + c) = (a \cdot b) + (a \cdot c)$.

   Using *only* the above facts and what you know about $=$, prove that $-1 \cdot -1 = 1$.