

# Math 22 Fall 2013

## Homework 1: Due on Wed Sep 25

Show all your calculations. You can receive partial credit for partially correct work, even if the final solution is incorrect. Therefore, spell out step-by-step calculations, and explain your answers to open questions.

1. (a) Solve the following system of linear equations:

$$\begin{array}{rccccrcr} & & & x_3 & + & 2x_4 & = & 7 \\ -2x_1 & - & 8x_2 & + & x_3 & + & 6x_4 & = & 17 \\ & x_1 & + & 4x_2 & + & x_3 & + & 3x_4 & = & 14 \end{array}$$

Show the steps of your calculation and give a *parametric description* of your solution.

- (b) There are infinitely many solutions. Give *one* specific solution, i.e., give an example of a list of specific numbers  $x_1, x_2, x_3, x_4$  that solves the linear system.
2. For which value(s) of the coefficient  $\mathbf{a}$  does the linear system below have *infinitely many* solutions?

$$\begin{array}{rccccrcr} & x_1 & & & & +\mathbf{a}x_3 & = & 3 \\ 3x_1 & + & 2x_2 & + & 3x_3 & = & 6 \\ 2x_1 & + & 2x_2 & + & 5x_3 & = & 3 \end{array}$$

Show the row operations that you performed, and explain in words why your value(s) for  $\mathbf{a}$  leads to infinitely many solutions.

3. Consider the following three vectors in  $\mathbb{R}^3$ :

$$\mathbf{u} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \mathbf{v} = \begin{pmatrix} 1 \\ 3 \\ -1 \end{pmatrix}, \mathbf{w} = \begin{pmatrix} 1 \\ 0 \\ 11 \end{pmatrix}$$

Is the vector  $\mathbf{w}$  a linear combination of  $\mathbf{u}$  and  $\mathbf{v}$ ? Show how you arrived at your answer.

4. True or False?

*For True/False questions you do not have to justify your answer!*

- (a) Two matrices are row equivalent if they have the same number of rows.
- (b) A consistent system has one or more solutions.
- (c) If every column of an augmented matrix contains a pivot, then the corresponding system is consistent.
- (d) A consistent system of 3 equations in 5 variables always has free variables.
- (e) A system of 5 equations in 3 variables is never consistent.