Math 22 Fall 2013

Problem set 4: Due on Wed Oct 16

Show all your calculations. You can receive partial credit for partially correct work, even if the final solution is incorrect. Therefore, spell out step-by-step calculations, and explain your answers to open questions.

1. (a) Calculate the inverse matrix A^{-1} of

$$A = \begin{pmatrix} 3 & -4 & 2 & 1 \\ 3 & 0 & 1 & 0 \\ 0 & 2 & 0 & -1 \\ 1 & -1 & 1 & 0 \end{pmatrix}$$

   (b) Use the inverse matrix A^{-1} to find the unique solution of the linear system below without doing more row reductions.

$$\begin{array}{rcrcrcrcl} 3x_1 & - & 4x_2 & + & 2x_3 & + & x_4 & = & 1 \\ 3x_1 & & & + & x_3 & & & = & 2 \\ & & 2x_2 & & & - & x_4 & = & 3 \\ x_1 & - & x_2 & + & x_3 & & & = & 4 \end{array}$$

2. Consider the matrix

$$B = \begin{pmatrix} 2 & 4 & 2 & 13 & 2 \\ 1 & 2 & 0 & 4 & -2 \\ 2 & 4 & -1 & 8 & -2 \\ 1 & 2 & -1 & 3 & -2 \end{pmatrix}$$

   (a) Find a basis for Nul$B$, which is a subspace of $\mathbb{R}^5$.

   (b) Find a basis for Col$B$, which is a subspace of $\mathbb{R}^4$.

3. (a) Explain in your own words why it is true that if Nul$A$ is just the point $\{\mathbf{0}\}$, then the solution of an equation $A\mathbf{x} = \mathbf{b}$ has at most one solution.

   (b) Explain in your own words why it is true that if Col$A$ is all of $\mathbb{R}^m$ (for a $m \times n$ matrix A) then $A\mathbf{x} = \mathbf{b}$ has at least one solution for every possible $\mathbf{b}$.

4. *For True/False questions you do* not *have to justify your answer!*

   (a) If A is a 7×7 matrix with 6 pivots then Nul$A$ has dimension 6.

   (b) If A is an invertible matrix, then $(A^T)^{-1}$ and $(A^{-1})^T$ are the same matrix.

   (c) If A is an $n \times n$ matrix, and there exists another square matrix B with $AB = I$, then the equation $A\mathbf{x} = \mathbf{b}$ has at least one solution for every possible $\mathbf{b}$ in $\mathbb{R}^n$.

   (d) If A and B are invertible matrices, then the product AB is also invertible.

   (e) Given vectors $\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_p$ in $\mathbb{R}^n$, the set of all linear combinations of these vectors is a subspace of $\mathbb{R}^n$.