

# Math 22 Fall 2013

## Problem set 5: Due on Wed Oct 23

Show all your calculations. You can receive partial credit for partially correct work, even if the final solution is incorrect. Therefore, spell out step-by-step calculations, and explain your answers to open questions.

1. (a) Consider the set  $\mathcal{B}$  of three vectors

$$\mathcal{B} = \left\{ \begin{pmatrix} 4 \\ 6 \\ 9 \end{pmatrix}, \begin{pmatrix} 3 \\ 5 \\ 7 \end{pmatrix}, \begin{pmatrix} 0 \\ 2 \\ 3 \end{pmatrix} \right\}$$

Verify that  $\mathcal{B}$  is a *basis* for  $\mathbb{R}^3$ , and explain how you got your answer!

- (b) If  $\mathbf{x} = \begin{pmatrix} 9 \\ 11 \\ 19 \end{pmatrix}$  is a column vector in  $\mathbb{R}^3$ , then what are its  $\mathcal{B}$ -coordinates  $[\mathbf{x}]_{\mathcal{B}}$ ?

2. Calculate the determinant of the matrix  $A$ ,

$$A = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 2 & 3 & 4 & 5 \\ 3 & 3 & 3 & 4 & 5 \\ 4 & 4 & 4 & 4 & 5 \\ 5 & 5 & 5 & 5 & 5 \end{pmatrix}$$

3. Let  $S$  be the tetrahedron in  $\mathbb{R}^3$  with vertices at the vectors  $\mathbf{0}$ ,  $\mathbf{e}_1$ ,  $\mathbf{e}_2$ ,  $\mathbf{e}_3$ , and let  $S'$  be the tetrahedron with vertices at the vectors  $\mathbf{0} = (0, 0, 0)$ ,  $\mathbf{v}_1 = (3, 1, 1)$ ,  $\mathbf{v}_2 = (1, 3, 1)$ ,  $\mathbf{v}_3 = (1, 1, 3)$ . See the picture for Exercise 32 in section 3.3 (p. 185).

- (a) What is the standard matrix  $A$  of the linear transformation  $T: \mathbb{R}^3 \rightarrow \mathbb{R}^3$  that maps  $S$  onto  $S'$ ?
- (b) The volume of the tetrahedron  $S$  is  $\frac{1}{6}$ . What is the volume of the tetrahedron  $S'$ ?