Math 22 Fall 2013

Problem set 7: Due on Wed Nov 13

Show all your calculations. You can receive partial credit for partially correct work, even if the final solution is incorrect. Therefore, spell out step-by-step calculations, and explain your answers to open questions.

1. (a) Find the least squares solution(s) of the equations $A\mathbf{x} = \mathbf{b}$, where

$$A = \begin{pmatrix} 2 & 3 \\ 2 & 4 \\ 1 & 1 \end{pmatrix}, \ \mathbf{b} = \begin{pmatrix} 7 \\ 3 \\ 1 \end{pmatrix}$$

   (b) Calculate the *least-squares error* of your solution.

2. We want to fit a parabola of the form $y = ax + bx^2$ to the following collection of data points (x, y) :

$$(0,0),\ (1,3),\ (1,4)\,(2,3),\ (4,1)$$

   Find the best least-squares approximation to these data of the form $y = ax + bx^2$.

   (**Note.** There are two data points for $x = 1$. The method still works.)

3. Find a diagonal matrix D and an *orthogonal* matrix P such that $A = PDP^{-1}$ for the matrix

$$A = \begin{pmatrix} 5 & 2 \\ 2 & 2 \end{pmatrix}$$